

1. (4pts) Convert to scientific notation or a decimal number:

$$\underbrace{34789.31}_{4 \text{ places}} = 3.478931 \times 10^4 \quad \underbrace{0.0003989}_{4 \text{ places}} = 3.989 \times 10^{-4} = 0.0003989$$

2. (14pts) Use formulas to expand:

$$(3y + 7)^2 = (3y)^2 + 2 \cdot 3y \cdot 7 + 7^2 = 9y^2 + 42y + 49$$

$$(5u + v^2)(5u - v^2) = (5u)^2 - (v^2)^2 = 25u^2 - v^4$$

$$\begin{aligned} (x + 5)^3 &= x^3 + 3 \cdot x^2 \cdot 5 + 3 \cdot x \cdot 5^2 + 5^3 \\ &= x^3 + 15x^2 + 75x + 125 \end{aligned}$$

3. (13pts) Simplify, showing intermediate steps.

$$\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$$

$$\sqrt[3]{-432} = \sqrt[3]{-2 \cdot 216} = \sqrt[3]{-2 \cdot 6 \cdot 6 \cdot 6} = -6 \sqrt[3]{2}$$

$$16^{\frac{5}{4}} = \left(\sqrt[4]{16}\right)^5 = 2^5 = 32$$

$$\begin{aligned} \sqrt{75x^9y^6} &= \sqrt{3 \cdot 25x^9 \cdot x \cdot y^6} = \sqrt{3 \cdot 25 \cdot (x^9)^2 \cdot x \cdot (y^3)^2} \\ &= 5x^4y^3\sqrt{3x} \end{aligned}$$

4. (8pts) Simplify.

$$\frac{4x+3}{x^2+3x-28} - \frac{2}{x^2-16} = \frac{4x+3}{(x+7)(x-4)} - \frac{2}{(x-4)(x+4)}$$

prod = -28 7, -4
sum = 3

$$= \frac{(4x+3)(x+4) - 2(x+7)}{(x+7)(x-4)(x+4)}$$

$$= \frac{4x^2 + 19x + 12 - 2x - 14}{(x+7)(x-4)(x+4)} = \frac{4x^2 + 17x - 2}{(x+7)(x-4)(x+4)}$$

prod = -8
sum = 17

-1, 8	-2, 4
±7	±2

does not factor

5. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{4(9x^{-4}y)^{\frac{3}{2}}}{(2x^{-5}y^{\frac{3}{8}})^4} = \frac{4 \cdot 9^{\frac{3}{2}} x^{-4 \cdot \frac{3}{2}} y^{\frac{3}{2}}}{2^4 \cdot x^{-20} y^{\frac{3}{8} \cdot 4}} = \frac{(\sqrt{9})^3 x^{-6} y^{\frac{3}{2}}}{4 x^{-20} y^{\frac{3}{2}}} = \frac{27 x^{-6+20} y^{\frac{3}{2}-\frac{3}{2}}}{4}$$

$$= \frac{27x^{14}}{4y}$$

6. (6pts) Rationalize the denominator.

$$\frac{3+2\sqrt{5}}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{12-3\sqrt{5}+8\sqrt{5}-2\sqrt{5}^2}{4^2-\sqrt{5}^2} = \frac{2+5\sqrt{5}}{16-5} = \frac{2+5\sqrt{5}}{11}$$

7. (11pts) Put the complex number into form $a + bi$.

$$\frac{3+2i}{3-5i} \cdot \frac{3+5i}{3+5i} = \frac{9+15i+6i+10i^2}{3^2-(5i)^2} = \frac{-1+21i}{9+25} = \frac{-1+21i}{34}$$

(explain) $i^{171} = i^{4 \cdot 42 + 3} = (i^4)^{42} \cdot i^3 = i^3 = -i$

$$171 \div 4 = 42, \text{ rem } 3$$

$$171 = 4 \cdot 42 + 3$$

8. (13pts) Solve the equations.

$$\frac{2x+1}{5} + 4 = \frac{3x+2}{6} - \frac{2x-7}{10} \quad | \cdot 30$$

$$\frac{2x+1}{5} \cdot 30 + 4 \cdot 30 = \frac{3x+2}{6} \cdot 30 - \frac{2x-7}{10} \cdot 30$$

$$12x+6 + 120 = 15x+10 - (6x-21)$$

$$12x+126 = 9x+31 \quad | -9x, -126$$

$$3x = -95$$

$$x = -\frac{95}{3}$$

$$\frac{3x-4}{x-2} + 4 = \frac{2}{x-2} \quad | \cdot (x-2)$$

$$3x-4 + 4(x-2) = 2$$

$$7x-12 = 2 \quad | +12, \div 7$$

$$7x = 14$$

$x=2$ ← not a solution to the original equation (gives 0 in denom.)

9. (12pts) How many liters of pure anti-freeze needs to be added to 5 liters of a 15% solution of antifreeze in order to get a 50% solution? Write down the meaning of the variable you use.

$x =$ amount of antifreeze needed

$$\left(\frac{x}{100\%} \right) + \left(\frac{5}{15\%} \right) = \left(\frac{x+5}{50\%} \right)$$

$$x + 0.15 \cdot 5 = 0.5(x+5)$$

$$x + 0.75 = 0.5x + 2.5 \quad | -0.5x, -0.75$$

$$0.5x = 1.75$$

$$x = \frac{1.75}{0.5} = 3.5 \text{ l}$$

10. (12pts) Ashley deposited some money in an account yielding 7%, and then deposited the same amount, and \$700 extra, in an account yielding 11%. How much did she deposit in each account, if after 9 months the interest earned from both accounts totaled \$408.75? Write down the meaning of the variable you use.

x = amount deposited in account yielding 7%

$$\text{interest 1} + \text{interest 2} = 408.75$$

$$x \cdot 0.07 \cdot \frac{9}{12} + (x+700) \cdot 0.11 \cdot \frac{9}{12} = 408.75$$

$$0.0525x + 0.0825(x+700) = 408.75$$

$$0.0525x + 0.0825x + 57.75 = 408.75$$

$$0.135x = 351$$

$$x = \frac{351}{0.135} = 2600$$

She put 2600 in account @ 7%
3300 in account @ 11%

Bonus (10pts) An oil tanker can be emptied by the main pump in 4 hours. An auxiliary pump can empty the tanker in 9 hours. If the main pump is started now, when should the auxiliary pump be turned on so that the tanker is emptied in exactly 3 hours? (Hint: Consider what portion of the job is to be done by the auxiliary pump.)

In one hour, main pump empties $\frac{1}{4}$ of tank.

" auxiliary " " " $\frac{1}{9}$ of tank

If the main pump works 3 hours, it empties $\frac{3}{4}$ of the tank. The auxiliary pump has to handle the remaining $\frac{1}{4}$ of tank.

T = hours need to empty $\frac{1}{9}$ of tank.

$$T \cdot \frac{1}{9} = \frac{1}{4}$$

$$T = \frac{9}{4} = 2\frac{1}{4}$$

The auxiliary pump needs to get started $2\frac{1}{4}$ hrs before targeted end, so after $\frac{3}{4}$ of an hour,