1. (12pts) Find all vectors whose angles with $\mathbf{i}$ and $\mathbf{k}$ are $\frac{\pi}{3}$ and $\frac{\pi}{4}$, respectively. *(Hint: look for unit vectors with unknown coordinates $\langle a, b, c\rangle$).*

2. (18pts) a) Find three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ for which $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$. (Use really simple vectors.)

b) Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be any vectors. Note that $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ is a vector that is in the plane spanned by $\mathbf{v}$ and $\mathbf{w}$, since it is perpendicular to $\mathbf{v} \times \mathbf{w}$, which is a normal vector for the plane spanned by $\mathbf{v}$ and $\mathbf{w}$. Therefore, $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ can be written as $\beta\mathbf{v} + \gamma\mathbf{w}$. Use coordinates to show that $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$.

3. (12pts) Let a cube be positioned so that one of its vertices is at the origin and three of its edges are along the positive x, y and z- axes. Let $A = (1, 1, 1)$ be a vertex of the cube. Use projection of vectors to find the distance from the vertex $B = (0, 0, 1)$ to the diagonal OA.

4. (18pts) Two parallel lines are given parametrically: $x = 1 - t$, $y = 4 + 2t$, $z = 3 + 2t$ and $x = 2t$, $y = 1 - 4t$, $z = -3 - 4t$.
Find the distance between those lines in two ways (you'd better get the same answer!):
a) Use the height and area of a parallelogram (draw a picture).
b) Find a plane perpendicular to the two lines that passes through a known point on one line. The intersection of the plane with the other line will give you another point. Find the distance between those points.

1. (16pts) Consider the curve given by $\mathbf{r}(t) = \langle \cos t, \sin(8t), \sin t \rangle$, $t \in [0, 2\pi]$.
a) Sketch this curve.
b) Find all parameters t at which the tangent line is parallel to the xz-plane.

2. (14pts) Using coordinates, prove the product rule for cross products: $(\mathbf{u} \times \mathbf{v})' = \mathbf{u}' \times \mathbf{v} + \mathbf{u} \times \mathbf{v}'$.

3. (18pts) A ball is launched from the origin with initial velocity $\langle 4, 3, 7\rangle$ and it bounces off the wall represented by the plane $x = 2$. Assuming it moves under the influence of gravity (set $g = 10$), with air resistance ignored, where is it at time $t = 2$? *(Hint: this amounts to solving two initial value problems.)*

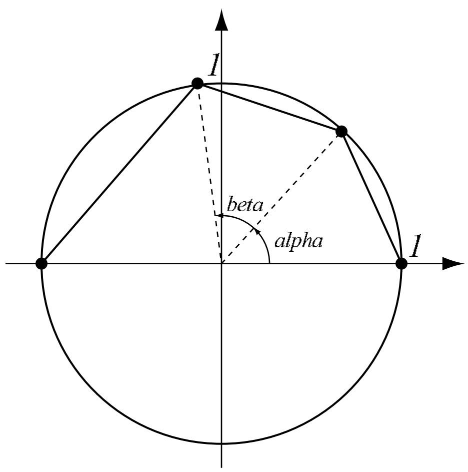
4. (12pts) Find and sketch the domain of the function $f(x, y, z) = \dfrac{\sqrt{9 - y^2 - z^2}}{\ln(x + 2y - 3z + 5)}$.

Fall '10/MAT 309/Worksheet 3 Name: *Show all your work.*

1. (20pts) An unfortunate twist of events finds you near the bottom of a snake pit described by the surface $z = \frac{x^2}{4} + \frac{y^2}{9} - 10$. You are at point $(2, 3, -8)$ and there is a caucus of rattlesnakes around the point $(0, 0, -10)$. You wish to move away from the snakes by taking the path of steepest ascent. Note that this is really a two-dimensional problem. Let $f(x, y) = \frac{x^2}{4} + \frac{y^2}{9} - 10$.
a) Draw the level curves of f for levels $c = -9, -8, -6, -1$.
b) From point $(2, 3)$, in which direction should you start going to achieve the greatest increase of f? Using the level curves you drew in a), draw an approximate path that always goes in the direction of greatest increase (this is the projection of your escape path to the xy plane).
c) Show that the path given by $\mathbf{c}(t) = \langle 2t^9, 3t^4 \rangle$ is such a path, that is, show that for every t, $\mathbf{c}'(t)$ is parallel to $\nabla f(\mathbf{c}(t))$.

2. (20pts) Let $x = r\cos\theta$, $y = r\sin\theta$ and let $f(x, y)$ be a function. Express $\frac{\partial^2 f}{\partial r^2}$, $\frac{\partial^2 f}{\partial\theta\partial r}$ and $\frac{\partial^2 f}{\partial\theta^2}$ in terms of r, θ and first and second partial derivatives of f by x and y. It's best if you do this on a separate sheet of paper, mwahahahahaha....

3. (20pts) Among all quadrangles inscribed in the unit circle whose one side is a diameter, find the one with the greatest area. Do it as follows:
a) Express the area A of the quadrangle as a function of angles α and β.
b) Determine the restrictions on α and β — this gives you the domain.
c) Find the maximum of A over the domain in the usual way.



1. (20pts) Find the centroid of the region that is common to circles $(x-3)^2+y^2=9$ and $x^2+(y-1)^2=1$. You can reduce your work by noticing that this region is symmetric about the line going through the center of the circles.

2. (20pts) Verify our change-of-variables formula for spherical coordinates by computing the Jacobian $\dfrac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)}$. What should you get?

3. (20pts) A spherical cap of height h is the set $x^2 + y^2 + z^2 \leq R^2$, $z \geq R - h$. Show that its volume is $V = \frac{1}{3}\pi h^2(3R - h)$. Then use this formula to get the volume of a ball or radius R. Cylindrical or spherical coordinates may be useful here.

Fall '10/MAT 309/Worksheet 5 **Name:** *Show all your work.*

For the following problems, you need to find the statements of the three major theorems in 17.1–17.3. Note the orientation conventions in those theorems.

1. (20pts) Verify Green's Theorem for the line integral $\oint_C xy\,dx + y\,dy$, where C is the unit circle, oriented clockwise (see section 17.1).

2. (20pts) Verify Stokes' Theorem for the part of the surface $z = 1 - x^2 - y^2$ where $z \geq 0$, where $\mathbf{F}(x, y, z) = \langle 2xy, x, y + z \rangle$ (see section 17.2). Use the upward pointing normal.

3. (20pts) Verify the Divergence Theorem for the sphere $x^2 + y^2 + z^2 = 4$ and the vector field $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$ (see section 17.3).