**Fall '10/MAT 309/Worksheet 2** **Name:** Solution *Show all your work.*

**1.** (16pts) Consider the curve given by $\mathbf{r}(t) = \langle \cos t, \sin(8t), \sin t\rangle$, $t \in [0, 2\pi]$.

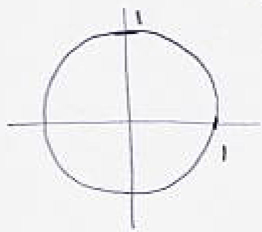
a) Sketch this curve.

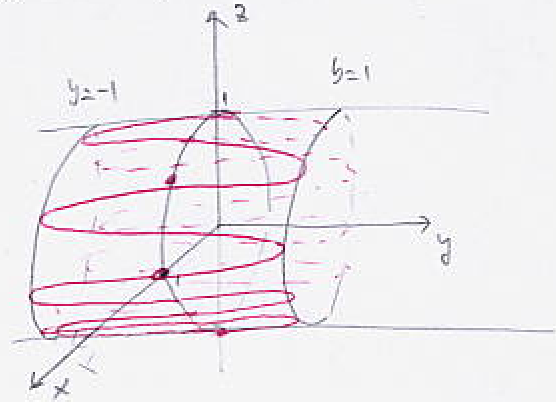
b) Find all parameters $t$ at which the tangent line is parallel to the $xz$-plane.

a) Projection to xz-plane is a circle, so curve is on cylinder $x^2+z^2=1$

$x = \cos t$

$z = \sin t$






Curve oscillates between $-1$ and $1$ in the y-direction, $-1 \le y \le 1$

b) $\vec{r}'(t) = \langle -\sin t, 8\cos 8t, \cos t\rangle$

$\vec{r}'(t)$ is parallel to xz-plane if its y-coord is zero

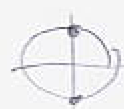
$8\cos(8t) = 0$  $8t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, \frac{29\pi}{2}, \frac{31\pi}{2}$

$\cos 8t = 0$  $t = \frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \dots, \frac{29\pi}{16}, \frac{31\pi}{16}$

$t$

$t \in [0, \frac{\pi}{2}]$

so $8t \in [0, 4\pi]$



**2.** (14pts) Using coordinates, prove the product rule for cross products: $(\mathbf{u} \times \mathbf{v})' = \mathbf{u}' \times \mathbf{v} + \mathbf{u} \times \mathbf{v}'$.

Let $\vec{u} = \langle a, b, c\rangle$, $\vec{v} = \langle d, e, f\rangle$ (all letters are functions of $t$)

$$(\vec{u}\times\vec{v})' = \frac{d}{dt}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{vmatrix} = \frac{d}{dt}\Big((bf-ce)\vec{i} - (af-cd)\vec{j} + (ae-bd)\vec{k}\Big)$$

$$= \big(b'f + bf' - (c'e + ce')\big)\vec{i} + \big(c'd + cd' - (a'f + af')\big)\vec{j} + \big(a'e + ae' - (b'd + bd')\big)\vec{k}$$

$$= (b'f - c'e + bf' - ce')\vec{i} + (c'd - a'f + cd' - af')\vec{j} + (a'e - b'd + ae' - bd')\vec{k}$$

these are equal

$$\vec{u}'\times\vec{v} + \vec{u}\times\vec{v}' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a' & b' & c' \\ d & e & f \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d' & e' & f' \end{vmatrix} =$$

$$= (b'f - c'e)\vec{i} - (a'f - c'd)\vec{j} + (a'e - b'd)\vec{k} + (bf' - ce')\vec{i} - (af' - cd')\vec{j} + (ae' - bd')\vec{k}$$

$$= (b'f - c'e)\vec{i} + (c'd - a'f)\vec{j} + (a'e - b'd)\vec{k} + (bf' - ce')\vec{i} + (cd' - af')\vec{j} + (ae' - bd')\vec{k}$$

**3.** (18pts) A ball is launched from the origin with initial velocity $\langle 4, 3, 7\rangle$ and it bounces off the wall represented by the plane $x = 2$. Assuming it moves under the influence of gravity (set $g = 10$), with air resistance ignored, where is it at time $t = 2$? *(Hint: this amounts to solving two initial value problems.)*

1) Motion until bounce. $(\vec{r}(t))$

$\vec{a}(t) = \langle 0, 0, -10\rangle$

$\vec{v}(t) = \langle 0, 0, -10t\rangle + \vec{C}$

$\langle 4,3,7\rangle = \vec{v}(0) = 0 + \vec{C}$

so $\vec{C} = \langle 4,3,7\rangle$

$\vec{v}(t) = \langle 4, 3, 7-10t\rangle$

$\vec{r}(t) = \langle 4t, 3t, 7t-5t^2\rangle + \vec{D}$

$\vec{0} = \vec{r}(0) = \vec{0} + \vec{D}$

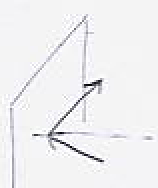
so $\vec{D} = \vec{0}$

$\vec{r}(t) = \langle 4t, 3t, 7t-5t^2\rangle$

Bounces from wall when $4t = 2$

$t = \frac{1}{2}$

Velocity vector after bounce: has x-coordinate reversed



$\vec{v}(\frac{1}{2}) = \langle 4,3,2\rangle$

2) Motion after bounce: $(\vec{r}_1(t))$

$\vec{r}_1(\frac{1}{2}) = \langle 2, \frac{3}{2}, \frac{9}{4}\rangle$

$\vec{v}_1(\frac{1}{2}) = \langle -4, 3, 2\rangle$

$\vec{a}_1(t) = \langle 0, 0, -10\rangle$

$\vec{v}_1(t) = \langle 0, 0, -10t\rangle + \vec{C}$

$\langle -4,3,2\rangle = \vec{v}_1(\frac{1}{2}) = \langle 0,0,-5\rangle + \vec{C}$

$\vec{C} = \langle -4, 3, 7\rangle$

$\vec{v}_1(t) = \langle -4, 3, 7-10t\rangle$

$\vec{r}_1(t) = \langle -4t, 3t, 7t-5t^2\rangle + \vec{D}$

$\langle 2, \frac{3}{2}, \frac{9}{4}\rangle = \vec{r}_1(\frac{1}{2}) = \langle -2, \frac{3}{2}, \frac{9}{4}\rangle + \vec{D}$

$\vec{D} = \langle 4, 0, 0\rangle$

$\vec{r}_1(t) = \langle 4-4t, 3t, 7t-5t^2\rangle$

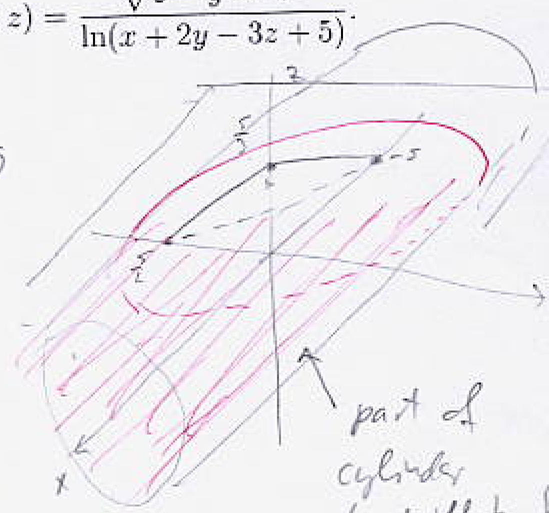
$\boxed{\vec{r}_1(2) = \langle -4, 6, -6\rangle}$

**4.** (12pts) Find and sketch the domain of the function $f(x, y, z) = \dfrac{\sqrt{9-y^2-z^2}}{\ln(x+2y-3z+5)}$.

Must have: $9 - y^2 - z^2 \geq 0$ — inside of solid cylinder $y^2+z^2 \leq 9$

$y^2 + z^2 \leq 9$

Must have $x + 2y - 3z + 5 > 0$ — space on one side of plane $x+2y-3z+5=0$ that includes origin




Note: Also should not have $x+2y-3z+5=1$ $(\ln 1 = 0)$

so exclude plane $x+2y-3z+4=0$ ⟶

but I did not see this at first