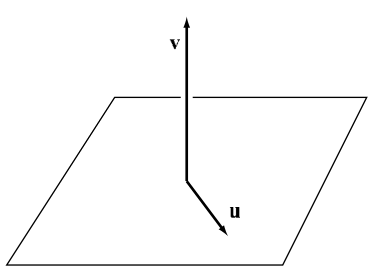
1. (22pts) Let $\mathbf{u} = \langle 1, 2. -3\rangle$ and $\mathbf{v} = \langle 4, -1, -4\rangle$.
a) Calculate $3\mathbf{u}$, $2\mathbf{u} - 4\mathbf{v}$, $\mathbf{u} \cdot \mathbf{v}$ and $||v||$.
b) Find the unit vector in direction of $\mathbf{v}$.
c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

2. (8pts) Vectors $\mathbf{u}$ and $\mathbf{v}$ are drawn below (they are perpendicular). Their lengths are $||\mathbf{u}|| = 3$ and $||\mathbf{v}|| = 1.5$. Draw the vector $\mathbf{u} \times \mathbf{v}$ and state its length.



3. (12pts) Find the point of intersection of the line $x = 2 + t$, $y = -3 + 2t$, $z = 5t$ with the plane $2x - 3y + z = 11$.

4. (20pts) Two lines are given parametrically: $x = 1 - t$, $y = 4 + 2t$, $z = 3 + 2t$ and $x = 2t$, $y = 1 - 4t$, $z = -3 - 4t$.
a) Show that these lines are parallel.
b) Find the equation of the plane spanned by these two lines.

5. (16pts) This problem is about the surface $-\left(\frac{x}{3}\right)^2+\left(\frac{y}{4}\right)^2-\left(\frac{z}{3}\right)^2=1.$

a) Identify and sketch the intersections of this surface with the coordinate planes.

b) Sketch the surface in 3D, with coordinate system visible.

6. (10pts) Sketch the following set of points given in cylindrical coordinates:

$\frac{\pi}{6}\leq\theta\leq\frac{\pi}{3},\ r>2$

7. (12pts) Sketch the point whose rectangular coordinates are $(-2, -2, \sqrt{\frac{8}{3}})$ and find its spherical coordinates.

Bonus (10pts) Refer to the parallel lines of problem 4.
a) Show that the lines are not identical. *(Hint: show a point on one line is not on the other.)*
b) Find the distance between those lines. *(Hints: one way uses the area of a parallelogram. Another uses a plane perpendicular to the lines.)*

1. (10pts) Write the parametrization of the circle that is the intersection of the sphere $x^2 + y^2 + z^2 = 16$ with the plane $x = 2$. Sketch a picture.

2. (20pts) A curve is given by $\mathbf{r}(t) = \langle 4t, t\cos t, t\sin t\rangle$, $t \in [0, 4\pi]$.
a) Sketch this curve.
b) Find the parametric equation of the tangent line to the curve at time $t = \pi$ and draw this tangent line on your sketch.

3. (22pts) After another ill-fated attempt at lunch, Wile E. Coyote finds himself ejected from the edge of a 60-meter tall canyon at angle $30°$ above the horizontal with initial speed 40 meters per second.

a) Find his position at time t. (For simplicity of calculation, blaspheme away and set $g = 10$.)

b) When does he hit the bottom of the canyon?

c) What is his speed when he hits the bottom?

4. (18pts) Find the length of the curve with the parametrization $\mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{\sqrt{3}} t^{\frac{3}{2}}, 3t + 7 \right\rangle$, $t \in [1, 5]$.

5. (20pts) Let $f(x, y) = x^2y$.
a) Identify and draw vertical traces for this function.
b) Using a), draw the graph of the function (in your 3-D coordinate system, orient the x-axis to the right, and the y-axis away from you).
c) Draw a rough contour map for the function, with contour interval 1, going from $c = -3$ to $c = 3$.
d) By looking at the contour map, indicate the direction (if any), in which we would have to move from $(1, 2)$ in order to decrease the value of the function.

6. (10pts) Determine and sketch the domain of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 9}$.

Bonus (10pts) Let $\mathbf{r}(t)$ the position of a moving object in space. If $\mathbf{r}'''(t) = \mathbf{0}$, use differentiation rules for products to help you show that the volume of the parallelepiped spanned by the position, velocity and acceleration vectors is constant. *(Hint: triple product.)*

1. (12pts) Find the equation of the tangent plane to the surface $x^2 - \frac{y^2}{4} + z^2 = 1$ at the point $\left(1, \sqrt{2}, \frac{\sqrt{2}}{2}\right)$. Simplify the equation to standard form.

2. (20pts) A bug is moving along the path $\mathbf{r}(t) = \langle 2t+3, t^2 \rangle$ through a region where temperature is distributed according to the function $T(x, y) = \dfrac{e^{x-y}}{x}$ (in °C).

a) Find the point P where the bug is at $t = 3$.

b) At what rate is the bug's temperature changing when $t = 3$ (in seconds)? What are the units?

c) At P, in which direction does the temperature decrease the fastest?

3. (20pts) Let $f(x, y) = x \ln(x^2 + y^2)$, $x = \sin u + \cos v$, $y = \cos u \sin v$. Find $\dfrac{\partial f}{\partial u}$ when $u = \pi$, $v = \frac{\pi}{2}$.

4. (12pts) A cylinder is measured to have radius $x = 20$cm and height $y = 12$cm, with an error in measurement at most 0.75cm in each. Estimate the maximal error in computing the volume of the cylinder.

5. (12pts) Find $\dfrac{\partial y}{\partial z}$ using implicit differentiation, if $x^2y + y^3z + z^4x = 13$.

6. (24pts) Find and classify the local extremes for the function $f(x, y) = x^2y^2 + y^4 - x^2 + 6y$.

Bonus (10pts) Consider the function $f(x, y) = x + \sqrt{3}y$ on the domain $x^2 + y^2 \leq 1$.
a) Determine the global minimum and maximum values of f.
b) Draw the graph of f and justify your answer from a) using the picture.

1. (18pts) Find $\iint_D y\, dA$ if D is the region bounded by the lines $y = 0$, $y = x$ and $y = 6 - x$. Sketch the region of integration.

2. (18pts) Evaluate $\displaystyle\int_0^1 \int_{2y}^2 y e^{x^3}\, dx\, dy$ by changing the order of integration. Sketch the region of integration.

3. (16pts) Use polar coordinates to evaluate the integral $\int_0^5 \int_0^{\sqrt{25-x^2}} (x+y)\,dy\,dx$. Sketch the region of integration first.

4. (16pts) Sketch the region W given by $x^2+y^2+z^2 \leq 9$, $z \geq 2$, $y \geq 0$. Then write the two iterated triple integrals that stand for $\iiint_W f\,dV$ which end in $dz\,dy\,dx$, and $dy\,dx\,dz$.

5. (16pts) Use cylindrical coordinates to set up $\iiint_W \frac{xyz}{x^2+y^2+1}\,dV$ where W is the region above the cone $z = \frac{1}{2}\sqrt{x^2+y^2}$, under the plane $z = 10$ and between the planes $y = x$ and $y = 0$ $(x, y \geq 0)$. Sketch the region of integration. Do not evaluate the integral.

6. (16pts) Use change of variables to find the integral $\iint_D e^{x-y}\,dA$ if D is the rectangle bounded by $y = x$, $y = x - 4$, $y = -x$ and $y = 8 - x$. Sketch the region D.

Bonus. (10pts) Use spherical coordinates to find the volume of the region W from problem 4.

1. (19pts) Let $\phi(x, y) = \sqrt{x^2 + y^2}$.
a) Find $\nabla\phi(x, y)$. What is $||\nabla\phi(x, y)||$?
b) Roughly draw the vector field $\nabla\phi(x, y)$, scaling the vectors for a better picture.
c) How could you have roughly done b) without the actual computation in a)?
d) What is $\int_C \nabla\phi \cdot d\mathbf{s}$ if C is part of the curve $y = x^3$ from $(0, 0)$ to $(2, 8)$? How about if C is a straight line segment from $(0, 0)$ to $(2, 8)$?

2. (15pts) Let C be part of the helix $x = 4\cos t$, $y = 4\sin t$, $z = t$, for $t \in [2\pi, 4\pi]$.
a) Set up $\int_C xz\, ds$.
b) Set up $\int_C \mathbf{F} \cdot d\mathbf{s}$, if $\mathbf{F}(x, y, z) = \langle y, x, z^2\rangle$.
In both cases simplify the set-up, but do not evaluate the integral.

3. (16pts) One of the two vectors fields below is not a gradient field, and the other one is (cross partials, remember?). Identify which is which, and find the potential function for the one that is.

$\mathbf{F}(x, y, z) = \langle x^2 + y^2, y^2 + z^2, z^2 + x^2 \rangle$ $\qquad\qquad$ $\mathbf{G}(x, y, z) = \langle e^z, e^z, e^z(x + y + z + 1) \rangle$

4. (24pts) Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if S is the part of the cylinder $x^2 + y^2 = 9$ between the planes $z = 1$ and $z = 5$, and $\mathbf{F}(x, y, z) = \langle -y, x - z, y \rangle$. (The surface does not include the top or the bottom, just part of the cylinder.) Use the normal vectors to the surface that point toward the z-axis. Draw the surface and some normal vectors, parametrize the surface and specify the planar region D where your parameters come from.

5. (26pts) Find the surface integral $\iint_S xz\, dS$, if S is part of the plane $4x + 2y + z = 4$ that is in the octant $x, y, z \geq 0$. Draw the surface (intercepts with the axes will help you draw the plane), parametrize it and specify the planar region D where your parameters come from.

Bonus. (10pts) A spherical cap (eek! It again!) of height h is the set $x^2 + y^2 + z^2 \leq R^2$, $z \geq R - h$. Show that its surface area is $A = 2\pi Rh$. Then use this formula to get the surface area of a ball or radius R.

1. (6pts) Let $\mathbf{u} = \langle 3, 1, -7 \rangle$ and $\mathbf{v} = \langle 2, -3, 1 \rangle$. Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

2. (16pts) The paraboloids $z = \frac{1}{2}(x^2 + y^2)$ and $z = 36 - x^2 - y^2$ intersect in a circle.
a) Sketch a picture.
b) Find a parametrization for the circle.
c) Find the parametric equation of the tangent line to the circle at point $(3\sqrt{2}, -\sqrt{6}, 12)$ and draw it on your sketch.

3. (10pts) A line is given parametrically: $x = 1 + 2t$, $y = -3 - t$, $z = 5 + 2t$. Find the equation of the plane that contains this line and the point $(-3, 7, 0)$.

4. (12pts) Let $f(x, y) = x^2 - y$.
a) Draw a rough contour map for the function with contour interval 1, going from $c = -3$ to $c = 3$.
b) Find ∇f and roughly draw this vector field. Note that no computation is needed to draw the vector field.
c) What is $\int_C \nabla f \cdot d\mathbf{s}$ if C is the arc of the unit circle that is in the first quadrant, going counterclockwise?

5. (16pts) Let $f(x,y) = ye^{xy}$, $x = u^3 - v^3$, $y = \frac{u}{v}$. Find $\dfrac{\partial f}{\partial v}$ when $u = 2$, $v = 1$.

6. (16pts) Find and classify the local extremes for the function $f(x,y) = x^3 - xy + y^3$.

7. (18pts) Find $\iint_D x\,dA$ if D is the region bounded by the curves $y = x^2 - 10$ and $y = 5x + 14$.

8. (16pts) Use cylindrical or spherical coordinates to set up $\displaystyle\iiint_W \frac{x^2+y^2+z^2}{x^2+y^2+1}\,dV$ where W is the region inside the sphere $x^2 + y^2 + z^2 \leq 16$, between the planes $y = \sqrt{3}x$ and $y = -\frac{1}{\sqrt{3}}x$, and where $y \geq 0$. Sketch the region of integration. Do not evaluate the integral.

9. (24pts) Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if S is the part of the cone $z = 2\sqrt{x^2 + y^2}$ for which $z \leq 12$, and $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$. Use the normal vectors to the surface that point upwards. Draw the surface and some normal vectors, parametrize the surface and specify the planar region D where your parameters come from.

10. (16pts) Sketch the region W given by $x^2 + y^2 \leq z \leq 25$, $x \geq y$, $x, y \geq 0$. Then write the two iterated triple integrals that stand for $\iiint_W f \, dV$ which end in $dz\,dx\,dy$, and $dx\,dz\,dy$.

Bonus. (15pts) A spherical cap of height h is the set $x^2 + y^2 + z^2 \leq R^2$, $z \geq R - h$. Show that its surface area is $A = 2\pi Rh$. Then use this formula to get the surface area of a ball or radius R.