

1. (7pts) Find the spherical coordinates of the point whose rectangular coordinates are $(-1, 2, 5)$.

2. (12pts) Let $z = x \sin(xy)$, $x = 8t - t^4$, $y = e^{\frac{1}{t}}$. Find $\frac{dz}{dt}$ when $t = 2$.

3. (10pts) Find parametric equations of the line that is the intersection of the planes $2x + y - 3z = 5$ and $x - y + 2z = 4$.

4. (10pts) Let $f(x, y) = y - x^3$.

a) Draw the level curves for f for the levels $k = -2, -1, 0, 1, 2$.

b) Roughly draw the vector field ∇f . Note that no computation is needed for this.

c) Compute $\int_C \nabla f \cdot d\mathbf{r}$, where C is the vertical line segment joining points $(1, -3)$ and $(1, 4)$.

d) If you are standing at the point $(4, -3)$, in which direction should you move to experience the greatest increase in f ?

5. (15pts) Find and classify the local extremes for the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

6. (16pts) Find $\iint_D \sin y^2 dA$ if D is the region bounded by the graph of $y = |x|$ and the line $y = 4$. Sketch the region of integration.

7. (14pts) Use either spherical or cylindrical coordinates to set up $\iiint_E z^2(x^2+y^2) dV$, where E is the region above the cone $z = \frac{1}{\sqrt{3}}\sqrt{x^2+y^2}$ and under the sphere $x^2 + y^2 + z^2 = 16$. Sketch the region of integration. Do not evaluate the integral.

8. (14pts) Sketch the region E bounded by the planes $z = 3$, $y = 0$, $z = 2x$ and the surface $y = \sqrt{x}$. Then write the iterated triple integral that stands for $\iiint_E f dV$ that ends in $dx dz dy$.

9. (22pts) Let D be the region between the curve $y = 4 - x^2$ and the x -axis and let C be its boundary, oriented in the positive (counterclockwise) direction.

a) Set up the two integrals needed to find $\int_C xy^2 dx + 2x^2y dy$ and evaluate the easy one.

b) Find $\int_C xy^2 dx + 2x^2y dy$ using Green's theorem.

10. (20pts) Let S be the part of the cylinder $y^2 + z^2 = 9$ that is between planes $x = 0$ and $x = 5$. Choose normal vectors for S so that they point away from the x -axis.

a) Write the parametric equations for this surface. Specify the planar region D where your parameters come from.

b) Use your parametrization to set up the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$.

c) Evaluate the integral from b).

Bonus (14pts) This problem is about the surface $x^2 + y^2 - z^2 = 1$.

a) Sketch and identify the intersections of this surface with the plane $z = k$.

b) Sketch the intersection of this surface with the rz -plane.

c) Use a) and b) to sketch the surface in 3D, with coordinate system visible.