Fall '07/MAT 309/Exam 4 **Name:** *Show all your work.*

1. (10pts) Roughly draw the vector field ∇f if $f(x, y) = y - x^2$. Note that it is possible to do this with no computation.

2. (10pts) Write the parametric equations for the part of the plane $z = 4 - y$ that lies inside of the cylinder $x^2 + z^2 = 10$. Specify the planar region D where your parameters come from.

3. (25pts) Let $\mathbf{F}(x,y,z) = 2xy\,\mathbf{i} + (x^2 + e^y \sin z)\,\mathbf{j} + e^y \cos z\,\mathbf{k}$ be a vector field.
a) Find $\operatorname{curl}\mathbf{F}$.
b) Is the field $\mathbf{F}$ conservative? If it is, find the function f so that $\nabla f = \mathbf{F}$.
c) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is any curve from $(2,-1,0)$ to $(3,1,\frac{\pi}{2})$.

4. (10pts) Let $\mathbf{F}(x,y,z) = \langle P, Q, R \rangle$. Show that $\operatorname{div}(f\mathbf{F}) = \nabla f \cdot \mathbf{F} + f \operatorname{div}\mathbf{F}$.

5. (25pts) Let D be the region inside the unit circle, and let C be its boundary, oriented clockwise. Evaluate the integral $\int_C xy^2\,dx + yx^2\,dy$ in two ways:
a) directly
b) using Green's theorem.

If you don't get the same answer in a) and b), write "-5" on the margin. (Just kidding! Go to next problem and then check back.)

6. (20pts) Set up the double integral for $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where the surface S is the part of the plane $x+y+2z=6$ in the first octant, and $\mathbf{F}(x,y) = \langle -y, x, x+y+z \rangle$. Use the downward-pointing normal vector. Carry out the set-up until you get iterated single integrals, but do not evaluate the integral.

Bonus (10pts) Find the surface area of the part of the sphere $x^2+y^2+z^2=R^2$ that lies above the plane $z=h$, $0 \leq h \leq R$.