

1. (16pts) Find $\iint_D y^2 dA$ if D is the region bounded by the lines $y = 0$, $y = -x$ and $y = \frac{1}{2} - \frac{x}{2}$. Sketch the region of integration.

2. (16pts) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{1+y^3} dy dx$ by changing the order of integration. Sketch the region of integration.

3. (10pts) Set up $\iint_D x \, dA$ in polar coordinates if D is the region inside the first-quadrant petal of the curve $r = \sin 2\theta$ that is also above the line $y = x$. Sketch the region, but do not evaluate the integral.

4. (12pts) Sketch the region whose volume is given by the triple integral below:

$$\int_0^1 \int_{-\sqrt{2-x}}^{\sqrt{2-x}} \int_0^{4-4y} 1 \, dz \, dy \, dx$$

5. (16pts) Use cylindrical coordinates to set up $\iiint_E xyz^2 dV$ where E is the region above the paraboloid $z = \frac{1}{2}(x^2 + y^2)$, under the sphere $x^2 + y^2 + z^2 = 35$ and between the planes $y = \sqrt{3}x$ and $y = -\sqrt{3}x$. Sketch the region of integration. Do not evaluate the integral.

6. (16pts) Sketch the region E bounded by the planes $z = 0$, $x = 0$, $2x + y + z = 6$ and $y - 2z = 0$. Then write the iterated triple integral that stands for $\iiint_E f dV$ that ends in $dy dz dx$.

7. (14pts) Use change of variables to find the area of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

Bonus. (10pts) Consider the region below the paraboloid $z = \frac{1}{2}(x^2 + y^2)$, inside the sphere $x^2 + y^2 + z^2 = 35$, and above the xy -plane.

- a) Set up the triple integral for the volume of this region in spherical coordinates.
- b) Evaluate the integral, with final answer in exact form (not decimal!).