Fall '10/MAT 140/Joysheet 8 **Name:** Solution *Show all your work.*

1. (8pts) Evaluate without using the calculator:

$\log_8 256 = \frac{8}{3}$ $\log_7 \frac{1}{49} = -2$ $\log_9 3 = \frac{1}{2}$ $\log_c \sqrt[4]{c^3} = \frac{3}{4}$

$256 = 2^8 = (\sqrt[3]{8})^8 = 8^{\frac{8}{3}}$ $7^{?} = \frac{1}{49}$ $9^{?} = 3$ $c^{?} = c^{\frac{3}{4}}$

2. (4pts) Use your calculator to find $\log_3 15$ with accuracy 4 decimal places. Show how you obtained your number.

$\log_3 15 = \frac{\log 15}{\log 3} = 2.4650$

3. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_6 (216(x^2+5x-24)^3) = \log_6 216 + \log_6 ((x+8)(x-3))^3$

$= 3 + 3\log_6((x+8)(x-3)) =$

$= 3 + 3(\log_6(x+8) + \log_6(x-3)) = 3 + 3\log_6(x+8) + 3\log_6(x-3)$

$\ln \frac{x^3y^4}{x^{-2}y^8} = \ln(x^3y^4) - \ln(x^{-2}y^8) = \ln x^3 + \ln y^4 - (\ln x^{-2} + \ln y^8)$

$= 3\ln x + 4\ln y - (-2\ln x + 8\ln y)$

$= 5\ln x + 4\ln y$

4. (14pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{2}\log(49x^{\frac{3}{4}}) + \frac{1}{4}\log(16x^7) = \log(49x^{\frac{3}{4}})^{\frac{1}{2}} + \log(16x^7)^{\frac{1}{4}} = \log\left[\left(\underbrace{49^{\frac{1}{2}}}_{7} x^{\frac{3}{8}}\right)\cdot\left(\underbrace{16^{\frac{1}{4}}}_{2} x^{\frac{7}{4}}\right)\right]$

$= \log(14x^{\frac{17}{8}})$ $\frac{3}{8} + \frac{7}{4} = \frac{3+14}{8} = \frac{17}{8}$

$3\log_b(x^2+4x-21) - 2\log_b(x+7) - 5\log_b(x-3) =$

$= \log_b(x^2+4x-21)^3 - \log_b(x+7)^2 - \log_b(x-3)^5$

$= \log_b[(x+7)(x-3)]^3 - (\log_b(x+7)^2 + \log_b(x-3)^5)$

$= \log_b[(x+7)^3(x-3)^3] - \log_b[(x+7)^2(x-3)^5]$

$= \log_b \frac{(x+7)^3(x-3)^3}{(x+7)^2(x-3)^5} = \log_b \frac{x+7}{(x-3)^2}$

5. (3pts) Find the domain of $f(x) = \log_{17}(7 - 3x)$.

Must have $7-3x>0 \quad | +3x$

$7>3x \quad | \div 3$ Domain: $(-\infty, \frac{7}{3})$

$\frac{7}{3}>x$

6. (12pts) Solve the equations.

$8^{2x+1} = 64^{3x-4}$

$8^{2x+1} = (8^2)^{3x-4}$

$8^{2x+1} = 8^{6x-8}$

$2x+1 = 6x-8 \quad | -2x+8$

$9 = 4x$

$x = \frac{9}{4}$

$3^{2x} = 2^{3x-5} \quad | \ln$

$\ln 3^{2x} = \ln 2^{3x-5}$

$2x \ln 3 = (3x-5)\ln 2$

$2x \ln 3 = 3x\ln 2 - 5\ln 2 \quad | -3x\ln 2$

$2x\ln 3 - 3x\ln 2 = -5\ln 2$

$x(2\ln 3 - 3\ln 2) = -5\ln 2$

$x = -\frac{5\ln 2}{2\ln 3 - 3\ln 2} = \frac{5\ln 2}{3\ln 2 - 2\ln 3} = -29.4247$

7. (8pts) If you invest \$2,000 in an account bearing 3%, compounded monthly, how long will it take until there is \$3,000 in the account?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3000 = 2000\left(1+\frac{0.03}{12}\right)^{12t} \quad | \div 2000$

$1.5 = (1.0025)^{12t} \quad | \ln$

$\ln 1.5 = \ln (1.0025)^{12t}$

$\ln 1.5 = 12t \ln(1.0025) \quad | \div 12\ln(1.0025)$

$t = \frac{\ln 1.5}{12 \ln 1.0025} = 13.5324$ years