Fall '10/MAT 140/Joysheet 7 **Name:** Solution *Show all your work.*

1. (20pts) Let $f(x) = \sqrt{x+3}$, $g(x) = \dfrac{x+1}{x-2}$. Find the following (simplify where possible):

$(f \cdot g)(x) = \sqrt{x+3} \cdot \dfrac{x+1}{x-2} = \dfrac{(x+1)\sqrt{x+3}}{x-2}$

State the domain of $(f \cdot g)(x)$ = domain of $f \cap$ domain of $g = [-3,2) \cup (2,\infty)$

($x+3 \geq 0$, $x \geq -3$; $x-2 \neq 0$, $x \neq 2$)

$\dfrac{f}{g}(3) = \dfrac{f(3)}{g(3)} = \dfrac{\sqrt{6}}{\frac{4}{1}} = \dfrac{\sqrt{6}}{4}$

$(f \circ g)(-2) = f(g(-2)) = f\left(\dfrac{-1}{-4}\right) = \sqrt{\dfrac{1}{4}+3} = \sqrt{\dfrac{13}{4}} = \dfrac{\sqrt{13}}{2}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+3})$

$= \dfrac{\sqrt{x+3}+1}{\sqrt{x+3}-2}$

$(f \circ f)(x) = f(f(x)) = f(\sqrt{x+3})$

$= \sqrt{\sqrt{x+3}+3}$

2. (12pts) Let $g(x) = \dfrac{x+1}{2x-9}$. Find the formula for g^{-1}. Find the domain and range of g.

$y = \dfrac{x+1}{2x-9}$

$y(2x-9) = x+1$

$2xy - 9y = x+1 \quad | -x+9y$

$2xy - x = 9y+1$

$x(2y-1) = 9y+1$

$x = \dfrac{9y+1}{2y-1}$

$g^{-1}(y) = \dfrac{9y+1}{2y-1}$

Domain of g: can't have $2x-9=0$, $2x=9$, $x=\frac{9}{2}$

$\{x \mid x \neq \frac{9}{2}\} = (-\infty, \frac{9}{2}) \cup (\frac{9}{2}, \infty)$

Range of g = domain of g^{-1}: can't have $2y-1=0$, $y=\frac{1}{2}$

$\{y \mid y \neq \frac{1}{2}\} = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

3. (8pts) Consider the function $h(x) = \dfrac{x^2+4}{x^2+7}$. Find functions f and g so that $h(x) = f(g((x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

$g(x) = x^2$ $\quad f(x) = \dfrac{x+4}{x+7}$

$g(x) = x^2+1$ $\quad f(x) = \dfrac{x+3}{x+6}$

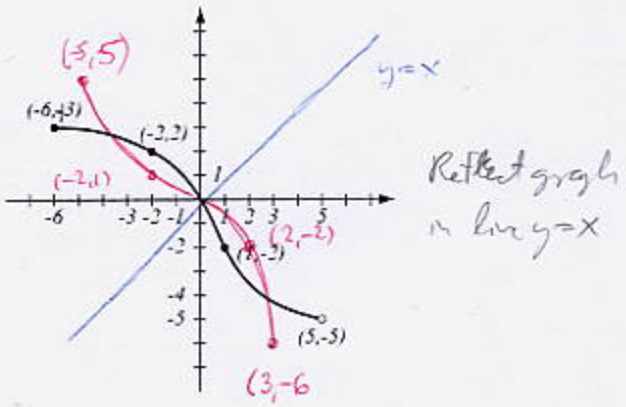
$g(x) = x^2+4$ $\quad f(x) = \dfrac{x}{x+3}$ $\quad$ etc,

4. (6pts) The graph of a function f is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.

a) It is one-to-one, since it passes the horizontal line test



5. (14pts) The quadratic function $f(x) = -x^2 + 3x - 1$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) y-int: $f(0) = -1$

x-int:

$-x^2+3x-1=0$

$x^2-3x+1=0$

$x = \dfrac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot1}}{2\cdot1}$

$= \dfrac{3\pm\sqrt{5}}{2} = 2.6180$ or 0.3820

b) $h = -\dfrac{b}{2a} = -\dfrac{3}{2\cdot(-1)} = \dfrac{3}{2}$

$k = -\left(\dfrac{3}{2}\right)^2 + 3\cdot\dfrac{3}{2} - 1$

$= -\dfrac{9}{4} + \dfrac{9}{2} - 1$

$= \dfrac{9}{4} - 1 = \dfrac{5}{4}$