1. (12pts) Use the graph of the function f at right to answer the following questions.
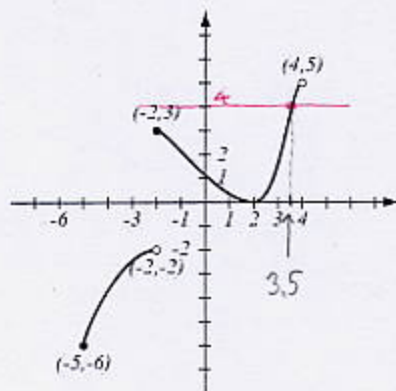
a) Find $f(4)$ and $f(-2)$. $f(4) = $ not defined, $f(-2) = 3$

b) What is the domain of f? $[-5, 4)$

c) What is the range of f? $[-6, -2) \cup [0, 5)$

d) What are the solutions of the equation $f(x) = 4$? $x = 3.5$

e) Find intervals where $f(x) > 0$. $[-2, 2) \cup (2, 5)$
(note: $f(2) = 0$)

2. (6pts) Find the domain of $f(x) = \dfrac{5x - 1}{x^2 + x - 20}$. Write your answer in interval notation.

$x^2 + x - 20 = 0$

$(x + 5)(x - 4) = 0$

$x = -5, 4$
$\underbrace{}$
must **not**
have this

$\xleftarrow{}\circ\underline{}\circ\xrightarrow{}$
$\quad -5 \quad\quad 4$

Domain $= \{x \mid x \neq -5, 4\}$

$\quad = (-\infty, -5) \cup (-5, 4) \cup (4, \infty)$
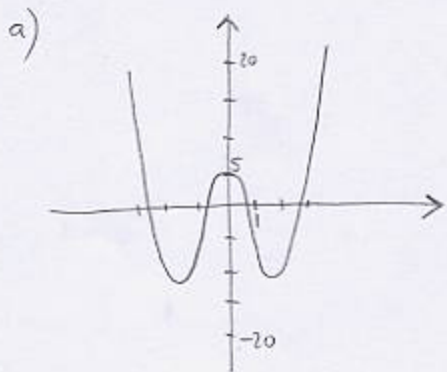
3. (14pts) Let $f(x) = x^4 - 9x^2 + 5$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
c) Find where f has a local minimum and maximum.
d) Find the intervals of increase and decrease.

a)

b) $f(-x) = (-x)^4 - 9(-x)^2 + 5 = x^4 - 9x^2 + 5 = f(x)$ So it is even

We see it also from the graph – it is symmetric w.r.t. y-axis

c) f has a local min. at $x = -2.1213$ whose value is $y = -15.25$

local max at $x = 0$ " $y = 5$

local min at $x = -2.1213$ " $y = -15.25$

d) increasing on $(-2.1213, 0) \cup (2.1213, \infty)$

decreasing on $(-\infty, -2.1213) \cup (0, 2.1213)$

4. (6pts) Let $f(x) = x^2 + 4x - 9$. For this function, calculate the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ and simplify.
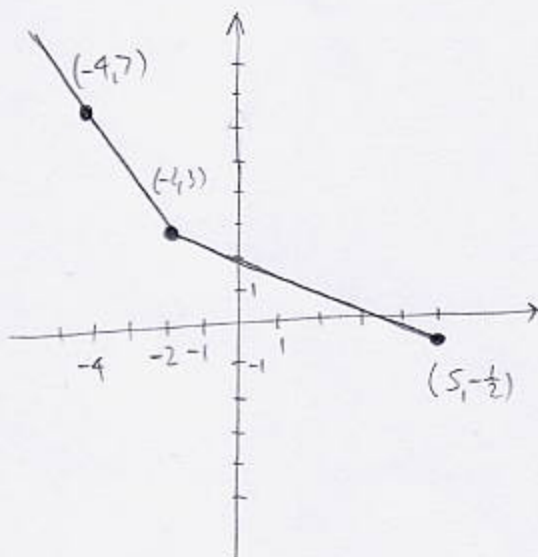
$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 4(x+h) - 9 - (x^2 + 4x - 9)}{h} = \frac{x^2 + 2xh + h^2 + 4x + 4h - 9 - x^2 - 4x + 9}{h}$$
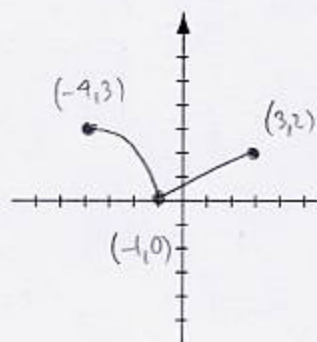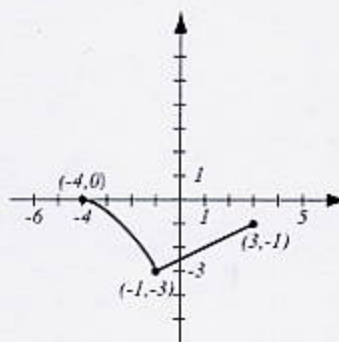
$$= \frac{2xh + 4h + h^2}{h} = 2x + 4 + h$$

5. (8pts) Sketch the graph of the piecewise-defined function:

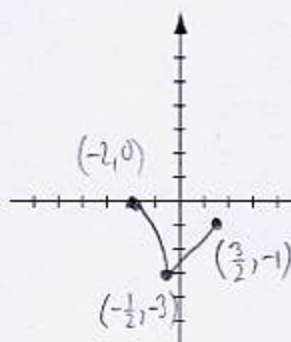$$f(x) = \begin{cases} -2x - 1, & \text{if } x < -2 \\ -\frac{1}{2}x + 2, & \text{if } -2 \le x \le 5. \end{cases}$$

x	$-2x-1$
-2	3
-4	7

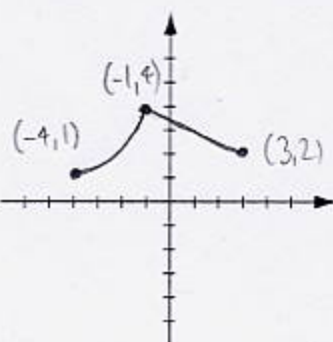| x	$-\frac{1}{2}x+2$
-2	3
5	$-\frac{1}{2}$

6. (14pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) + 3$, $f(2x)$ and $-f(x) + 1$ and label all the relevant points.

shift up 3

stretch horizontally by factor $\frac{1}{2}$

reflect in x-axis first, shift up 1