Fall '10/MAT 140/Joysheet 5 **Name:** Solution *Show all your work.*

1. (19pts) Let $A = (-3, 0)$, $B = (3, 0)$ and $C = (0, 3)$.
a) Is the triangle ABC right, isosceles or equilateral?
b) Verify that the points A, B and C are all on the circle $x^2 + y^2 = 9$. Draw the circle.
c) Find a point D on the upper half of the circle other than A, B or C. Draw D in your picture. Show that the triangle ABD is a right triangle.

a) $d(A,B) = \sqrt{(3-(-3))^2 + (0-0)^2} = \sqrt{6^2} = 6$

$d(A,C) = \sqrt{(0-(-3))^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

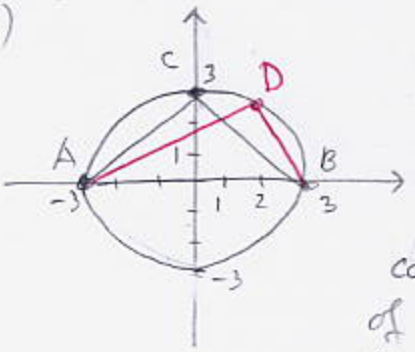
$d(B,C) = \sqrt{(0-3)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

ABC is not equilateral, but is isosceles

$\sqrt{18}^2 + \sqrt{18}^2 \stackrel{?}{=} 6^2$

$36 = 36$ yes, it is a right triangle

b)



$(-3)^2 + 0^2 = 9$

$3^2 + 0^2 = 9$

$0^2 + 3^2 = 9$

coordinates of A, B, C all satisfy the equation.

c) Choose an x for D, say $x = 2$

$2^2 + y^2 = 9$ $(2, \sqrt{5})$ is

$y^2 = 5$ on the upper

$y = \pm\sqrt{5}$ half-circle

$d(A,D) = \sqrt{(2-(-3))^2 + (\sqrt{5}-0)^2} = \sqrt{25+5} = \sqrt{30}$

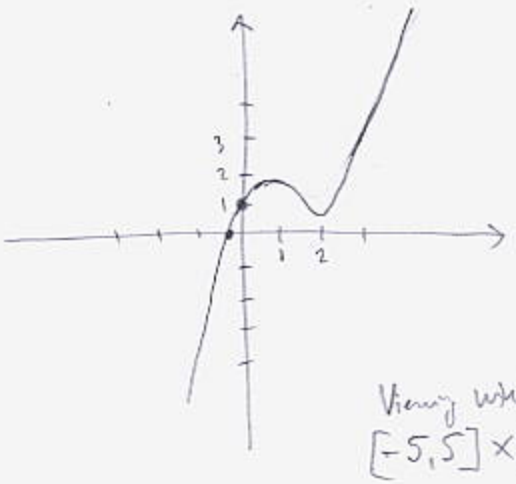
$d(B,D) = \sqrt{(2-3)^2 + (\sqrt{5}-0)^2} = \sqrt{1+5} = \sqrt{6}$

$d(A,B) = 6$ from a)

Check ABD is a right triangle: $\sqrt{30}^2 + \sqrt{6}^2 \stackrel{?}{=} 6^2$

$30 + 6 = 36$ yes

2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 4x^2 + 4x + 1$. Draw the graph here, and indicate the viewing window. Find all the $x-$ and $y-$intercepts (accuracy: 4 decimal points).



Viewing window: $[-5, 5] \times [-10, 10]$

y-int: $x = 0$, $y = 1$

x-int: $0 = x^3 - 4x^2 + 4x + 1$

Using calculator: $x = -0.2056$

3. (7pts) Find the equation of the line (in form $y = mx + b$) that passes through points $(-1, -2)$ and $(2, 3)$.

$m = \frac{3-(-2)}{2-(-1)} = \frac{5}{3}$

$y - 3 = \frac{5}{3}(x-2)$

$y = \frac{5}{3}x - \frac{10}{3} + 3$

$y = \frac{5}{3}x - \frac{1}{3}$

4. (14pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x - 4y = 5$, and passes through point $(-4, 0)$. Draw both lines.

$3x - 4y = 5 \quad | +4y - 5$

$3x - 5 = 4y \quad | \div 4$

$y = \frac{3}{4}x - \frac{5}{4}$

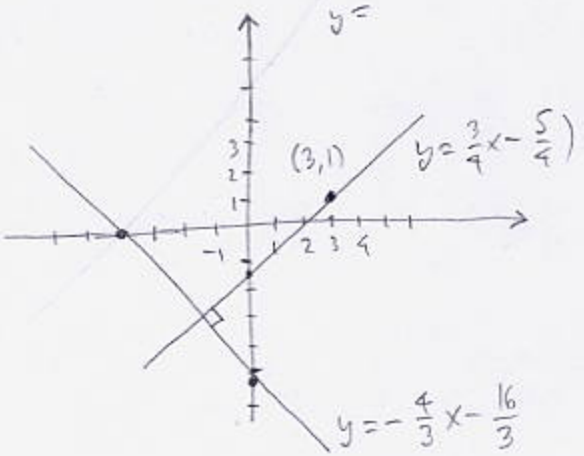
slope $= \frac{3}{4}$,

slope of perpendicular line is $-\frac{1}{\frac{3}{4}} = -\frac{4}{3}$

$y - 0 = -\frac{4}{3}(x-(-4))$

$y = -\frac{4}{3}x - \frac{16}{3}$

$\frac{16}{3} = 5\frac{1}{3}$



5. (10pts) The equation $x^2 + y^2 + 6x - 8y = 0$ represents a circle. Find its center and radius and draw the circle.

$x^2 + y^2 + 6x - 8y = 0$ $\quad 3 = \frac{6}{2}$ $\quad 4 = \frac{8}{2}$

$x^2 + 6x + y^2 - 8y = 0 \quad | +3^2 + 4^2$

$x^2 + 6x + 3^2 + y^2 - 8y + 4^2 = 25$

$(x+3)^2 + (y-4)^2 = 5^2$

$(x-(-3))^2 + (y-4)^2 = 5^2$

Center: $(-3, 4)$

radius: 5