Fall '10/MAT 140/Joysheet 4 **Name:** Solution *Show all your work.*

1. (23pts) Solve the equations.

$6x^2 - 6x + 4 = x^2 + x$ $\quad | -x^2 - x$

$5x^2 - 7x + 4 = 0$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4\cdot5\cdot4}}{2\cdot5}$

$= \dfrac{7 \pm \sqrt{49-80}}{10} = \dfrac{7 \pm \sqrt{-31}}{10}$

$= \dfrac{7 \pm i\sqrt{31}}{10}$

$2x + 4 = x - \sqrt{6x+51}$ $\quad | -x$

$x + 4 = -\sqrt{6x+51}$ $\quad |^2$

$x^2 + 8x + 16 = 6x + 51$ $\quad | -6x - 51$

$x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$x = -7, 5$

Test in original equation:

$2(-7)+4 \overset{?}{=} -7 - \sqrt{-42+51}$

$-10 \overset{?}{=} -7 - \sqrt{9}$ yes

$2\cdot5+4 \overset{?}{=} 5 - \sqrt{30+51}$

$14 \overset{?}{=} 5 - 9$ no

Only $x = -7$ is the solution.

$4x^4 - 11x^2 - 3 = 0$

Set $u = x^2$

$4u^2 - 11u - 3 = 0$ $\quad 121 + 48$

$u = \dfrac{-(-11) \pm \sqrt{(-11)^2 - 4\cdot4\cdot(-3)}}{2\cdot4} = \dfrac{11 \pm \sqrt{169}}{8}$

$= \dfrac{11 \pm 13}{8} = 3, -\frac{1}{4}$

$x^2 = 3 \qquad x^2 = -\frac{1}{4}$

$x = \pm\sqrt{3} \qquad x = \pm\frac{1}{2}i$

2. (6pts) Solve by completing the square.

$x^2 - 10x + 5 = 8$ $\quad | + 5^2 \quad \left(5 = \frac{10}{2}\right)$

$x^2 - 2\cdot x\cdot 5 + 5^2 + 5 = 8 + 25$ $\quad | -5$

$(x-5)^2 = 28$

$x - 5 = \pm\sqrt{28}$

$x = 5 \pm 2\sqrt{7}$

3. (4pts) Solve the equation.

$|3x - 1| = 13$

$3x-1=13$ or $3x-1=-13$

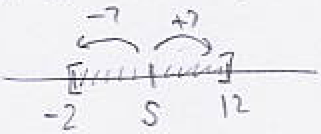
$3x=14$ $\quad$ $3x=-12$

$x=\frac{14}{3}$ or $x=-4$

4. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|2x - 5| \leq 7$

distance from $2x$ to $5 \leq 7$



$-2 \leq 2x \leq 12 \quad | \div 2$

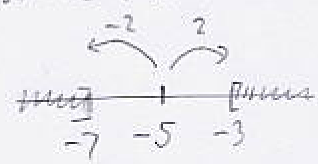
$-1 \leq x \leq 6$

$[-1, 6]$

$|x + 5| \geq 2$

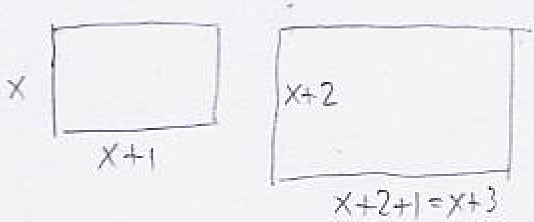
$|x-(-5)| \geq 2$

distance from x to $-5 \geq 2$



$(-\infty, -7] \cup [-3, \infty)$

5. (15pts) A landscaper plans to cover two rectangular areas with stone tiles, of which she has enough to cover 20 square feet. One of the rectangles has width 2 feet more than the other, and both rectangles have lengths that are 1 foot more than their respective widths. Assuming the landscaper uses up all the tiles, what are the dimensions of the rectangles?



Combined areas equal 20:

$x(x+1) + (x+2)(x+3) = 20$

$x^2 + x + x^2 + 5x + 6 = 20$

$2x^2 + 6x + 6 = 20 \quad | -20$

$2x^2 + 6x - 14 = 0 \quad | \div 2$

$x^2 + 3x - 7 = 0$ doesn't factor

$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-7)}}{2 \cdot 1} = \frac{-3 \pm \sqrt{9+28}}{2} = \frac{-3 \pm \sqrt{37}}{2}$

The negative solution $\frac{-3-\sqrt{37}}{2}$ does not fit context, since $x > 0$.

Rectangles: $\frac{-3+\sqrt{37}}{2}$ by $\frac{-3+\sqrt{37}}{2} + 1 = \frac{-1+\sqrt{37}}{2}$

$\frac{-3+\sqrt{37}}{2} + 2 = \frac{1+\sqrt{37}}{2}$ by $\frac{-3+\sqrt{37}}{2} + 3 = \frac{3+\sqrt{37}}{2}$