

1. (16pts) Simplify.

$$\frac{3x-2}{4x^2-17x+4} - \frac{5}{x^2+3x-28} = \frac{3x-2}{(4x-1)(x-4)} - \frac{5}{(x+7)(x-4)}$$

prod = 16    -16, -1  
sum = -17

$$= \frac{(3x-2)(x+7) - 5(4x-1)}{(4x-1)(x-4)(x+7)}$$

prod = -27    -27, 1    -9, 3  
sum = -1    no    no

does not factor

$$= \frac{3x^2+19x-14 - (20x-5)}{(4x-1)(x-4)(x+7)} = \frac{3x^2-x-9}{(4x-1)(x-4)(x+7)}$$

$$2 + \frac{4}{x-5} = \frac{2(x-5) + 4}{x-5} = \frac{2x-10+4}{x-5} = \frac{2x-6}{x-5} = \frac{2(x-3)}{(x-3)(x-2)} = \frac{2}{x-2}$$

$$x + \frac{6}{x-5} = \frac{x(x-5) + 6}{x-5} = \frac{x^2-5x+6}{x-5} = \frac{(x-3)(x-2)}{x-5}$$

2. (9pts) Simplify, showing intermediate steps.

$$\sqrt{108} = \sqrt{3 \cdot 36} = 6\sqrt{3}$$

$$125^{\frac{2}{3}} = (\sqrt[3]{125})^2 = 5^2 = 25$$

$$(-8)^{\frac{5}{3}} = (\sqrt[3]{-8})^5 = (-2)^5 = -32$$

3. (14pts) Simplify. Express answers in terms of positive exponents.

$$\sqrt[3]{48x^{21}y^{18}} = \sqrt[3]{8 \cdot 6 x^{21} y^{18}} \\ = 2 \sqrt[3]{6 x^7 y^6} = 2x^7 y^6 \sqrt[3]{6y}$$

$$\frac{(256x^2y^{-\frac{3}{2}})^{\frac{1}{4}}}{8x^{\frac{5}{4}}(27y^{\frac{3}{4}})^{-\frac{1}{3}}} = \frac{256^{\frac{1}{4}} x^{2 \cdot \frac{1}{4}} y^{-\frac{3}{2} \cdot \frac{1}{4}}}{8x^{\frac{5}{4}} 27^{-\frac{1}{3}} y^{\frac{3}{4} \cdot (-\frac{1}{3})}} = \frac{\sqrt[4]{256} x^{\frac{1}{2}} y^{-\frac{3}{8}} \cdot 27^{\frac{1}{3}}}{8x^{\frac{5}{4}} y^{-\frac{1}{4}}} = \frac{1}{8} x^{\frac{2}{2} - \frac{5}{4}} y^{-\frac{3}{8} - (-\frac{1}{4})} (\sqrt[3]{27})^2 \\ = \frac{x^{-\frac{3}{4}} y^{\frac{1}{8}} \cdot 3^2}{2} = \frac{9y^{\frac{1}{8}}}{2x^{\frac{3}{4}}}$$

4. (6pts) Rationalize the denominator.

$$\frac{\sqrt{5}-3}{1+2\sqrt{5}} = \frac{\sqrt{5}-3}{1+2\sqrt{5}} \cdot \frac{1-2\sqrt{5}}{1-2\sqrt{5}} = \frac{\sqrt{5}-2\sqrt{5}+3+6\sqrt{5}}{1-(2\sqrt{5})^2} = \frac{\sqrt{5}-10-3+6\sqrt{5}}{1-4 \cdot 5} = \frac{7\sqrt{5}-13}{-19} = \frac{13-7\sqrt{5}}{19}$$

5. (10pts) Simplify.

$$(2+5i)(3-2i) = 6 - 4i + 15i - 10i^2 = 16 + 11i$$

$$\frac{7+2i}{3-4i} = \frac{7+2i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{21+28i+6i+8i^2}{3^2-(4i)^2} = \frac{13+34i}{9-(-16)} = \frac{13+34i}{25}$$

6. (5pts) Simplify and justify your answer.

$$i^{259} = i^{256} \cdot i^3 = (i^4)^{64} \cdot i^3 = i^3 = -i$$

$$256 = 4 \cdot 64$$