

1. (4pts) Convert to scientific notation or a decimal number:

$$0.0003458 =$$

$$4.121 \times 10^7 =$$

2. (14pts) Use formulas to expand:

$$(2x - 3)(2x + 3) =$$

$$(a^2 + 3b)^2 =$$

$$(x - 4)^3 =$$

3. (14pts) Simplify, showing intermediate steps.

$$\sqrt{54} =$$

$$\sqrt[5]{64} =$$

$$27^{-\frac{4}{3}} =$$

$$\sqrt{48x^{12}y^5} =$$

4. (8pts) Simplify.

$$\frac{3}{x^2 + 2x - 15} - \frac{2}{x^2 + 7x + 10} =$$

5. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{(8x^6y^{-2})^{\frac{2}{3}}}{(6x^{-3}y^{\frac{5}{3}})^2} =$$

6. (6pts) Rationalize the denominator.

$$\frac{1 + 4\sqrt{3}}{2 - \sqrt{3}}$$

7. (11pts) Simplify, with answer in form  $a + bi$ .

$$(3 + 2i)(4 - 9i) + 4i(5 - i) =$$

$$\text{(explain) } i^{921} =$$

8. (12pts) Solve the equations.

$$\frac{3x + 2}{6} + 4 = -3x + 1 - \frac{2x - 1}{4}$$

$$\frac{5x + 3}{x + 1} - 7 = \frac{x}{x + 1}$$

9. (12pts) How many liters of a 15% solution of hydrochloric acid needs to be added to 100 liters of a 35% solution of hydrochloric acid in order to get a 20% solution? Write down the meaning of the variable you use.

**10.** (12pts) Fred, who is from Seattle, went to school in Florida. On the way to school, he took a southern route, and on his return after graduation, he took a northern route. On both trips he averaged the same speed. If the southern trek took 52 hours, the northern 60 hours, and the northern trek was 448 miles longer, how long was each trip? Write down the meaning of the variable you use.

**Bonus** (10pts) Solve the equation for a complex number  $z$ . Your answer must be in form  $a + bi$ . (*Hint: Don't fear this linear equation. Just do what you would ordinarily do, only with complex numbers.*)

$$\frac{iz + 1}{3 - 2i} + 4 = 5z + 3i$$

1. (6pts) Write the equations of the vertical and horizontal lines that pass through the point  $(3, -2)$ . Draw a picture.

2. (6pts) Are the lines  $2x - 3y = 5$  and  $y = -\frac{2}{3}x + 6$  parallel? Explain.

3. (5pts) Solve the equation.

$$|2x + 5| = 11$$

4. (14pts) Solve the inequalities and write the solution using interval notation:

$$4 - 3x > 7$$

$$|3x - 10| \geq 6$$

5. (15pts) Solve the equations.

$$2x^2 - x + 3 = 5 - 4x^2$$

$$x - 3 = \sqrt{33 - 8x}$$

6. (10pts) Below is an equation of a circle. Find the center and radius of the circle and draw the circle.

$$x^2 + y^2 + 8x + 12y - 12 = 0$$

**7.** (10pts) Use your calculator to accurately sketch the graph of  $y = -x^3 + 3x^2 + 7x + 4$ . Draw the graph here, and indicate the viewing window. Find all the  $x$ - and  $y$ -intercepts (accuracy: 4 decimal points).

**8.** (20pts) Let  $A = (4, -1)$ ,  $B = (1, 10)$  and  $C = (-1, 0)$ .

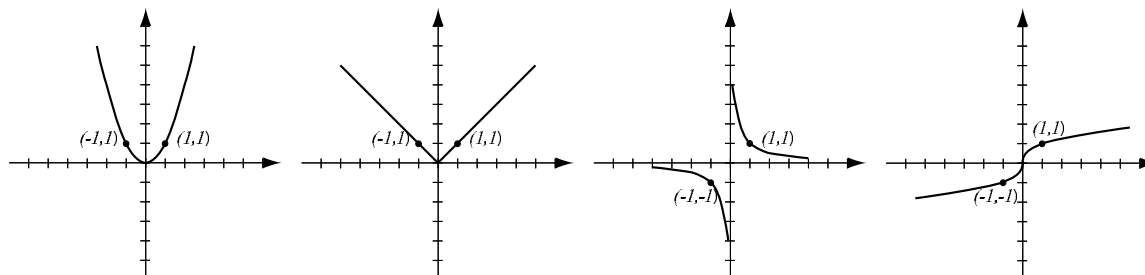
- a) Draw these points in the coordinate system.
- b) Show algebraically that the triangle  $ABC$  is a right triangle.
- c) Find the equation of the line that passes through  $C$  and is perpendicular to the side  $AB$ .
- d) Draw the line in the picture.

9. (14pts) Wilma has a rectangular picture whose area is 15 square feet. What are the dimensions of the picture if the width is 3 feet more than the height?

**Bonus** (10pts) Find the equation of the set of all points in the plane that have equal distance to  $(-3, 2)$  and  $(5, -4)$ . Draw the picture. You should get a line perpendicular to the line between the two points. (*Hint: Set up an equation involving the distance between a generic point  $(x, y)$  and the two points. Then simplify the equation until you get the equation of a line.*)

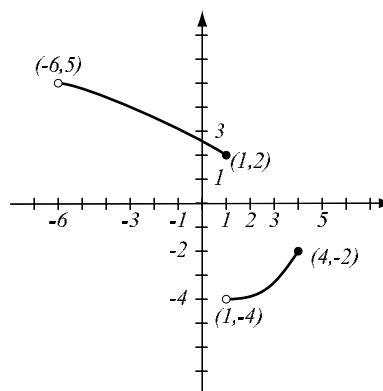


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (10pts) Use the graph of the function  $f$  at right to answer the following questions.

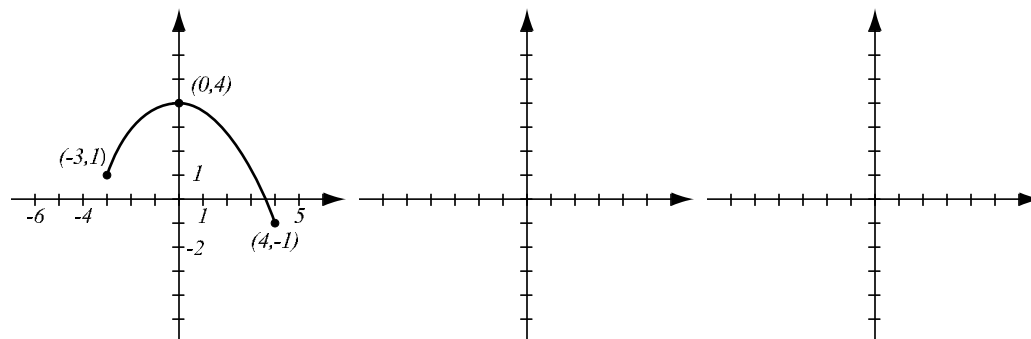
- Find  $f(4)$  and  $f(1)$ .
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = 3$ ?
- Find all  $x$  for which  $f(x) \geq 2$ .



3. (15pts) The quadratic function  $f(x) = x^2 + 2x - 15$  is given. Do the following without using the calculator.

- Find the  $x$ - and  $y$ -intercepts of its graph, if any.
- Find the vertex of the graph.
- Sketch the graph of the function.
- Is the function one-to-one? Justify.

4. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $f(-x) - 2$  and  $2f(x)$  and label all the relevant points.



5. (18pts) Let  $f(x) = \frac{x - 3}{x^2 + 3x - 4}$ ,  $g(x) = x + 2$ .

Find the following (simplify where possible):

$$\frac{f}{g}(2) =$$

$$(f \cdot g)(x) =$$

$$(g \circ f)(0)$$

$$(f \circ g)(x) =$$

The domain of  $(f - g)(x)$

**6.** (21pts) Let  $f(x) = x^3 - 13x$  (answer with 4 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of  $f$  (on paper!). Indicate scale on the graph.

b) Determine algebraically whether  $f$  is even, odd, or neither. Justify your answer further by examining the graph.

c) Algebraically find the  $x$ - and  $y$ -intercepts.

d) Find where  $f$  has a local minimum and maximum.

e) Find the intervals of increase and decrease.

**7.** (10pts) Let  $f(x) = x^2 + 3, x \geq 0$ .

a) Find the formula for  $f^{-1}$ .

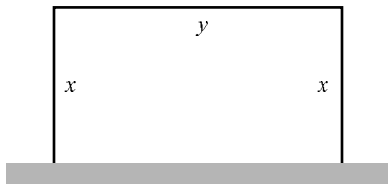
b) Find the range of  $f$ .

8. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 3, & \text{if } x \leq -1 \\ -x + 3, & \text{if } -1 < x. \end{cases}$$

**Bonus.** (10pts) Eric has a 50ft long fence that he will use to enclose a rectangular pen for his dog along a wall of his house (there is no fence along the wall). Follow the steps below to find the dimensions of the pen that has the greatest area.

- Write the area of the pen in terms of  $x$  and  $y$ . Then use the condition above to help you write the area  $A(x)$  as a function only of  $x$ .
- You should have gotten a quadratic function for  $A(x)$ . Graph it and determine algebraically where it achieves a maximum.
- What are the dimensions of the pen with the greatest area? What is the greatest area?



1. (8pts) Evaluate without using the calculator:

$$\log_5 25 = \quad \log_{1000} 10 = \quad \log_c \sqrt[6]{c^7} = \quad \log_{25} 125 = \quad (\text{think root})$$

2. (4pts) Use your calculator to find  $\log_7 3$  with accuracy 4 decimal places. Show how you obtained your number.

3. (13pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log((4y + 1)^5 \sqrt[3]{5x - 6}) =$$

$$\log_8 \left( \frac{\sqrt{y^3}}{64x^7} \right) =$$

4. (13pts) Write as a single logarithm. Simplify if possible.

$$4 \log(2x^3) - 3 \log(4x^5) =$$

$$3 \ln(x^2 - x - 30) + \ln(x + 5) - 2 \ln(x - 6) =$$

5. (8pts) How much should you invest in an account bearing 4.25%, compounded quarterly, if you wish to have \$4,000 in four years?

6. (8pts) Draw the graphs of the exponential function  $f(x) = b^x$  for  $0 < b < 1$  and for  $b > 1$ . What is the range in both cases?

7. (12pts) Let  $f(x) = -\log_2(x - 3)$ .

a) What is the domain of  $f$ ?

b) Use transformations to draw the graph of  $f$ , starting with the graph of  $y = \log_2 x$ . Indicate the  $x$ -intercept and any asymptotes.

Solve the equations.

8. (8pts)  $3^{x+2} = \left(\frac{1}{9}\right)^{2-x}$

9. (12pts)  $\log_9(x + 6) + \log_9(x + 4) = \log_9(3x + 12)$

**10.** (14pts) Five years ago, the population of Bunny City was 45,000, today, it is 57,000. Assume that the population grows according to the usual formula  $N(t) = N_0e^{rt}$ .

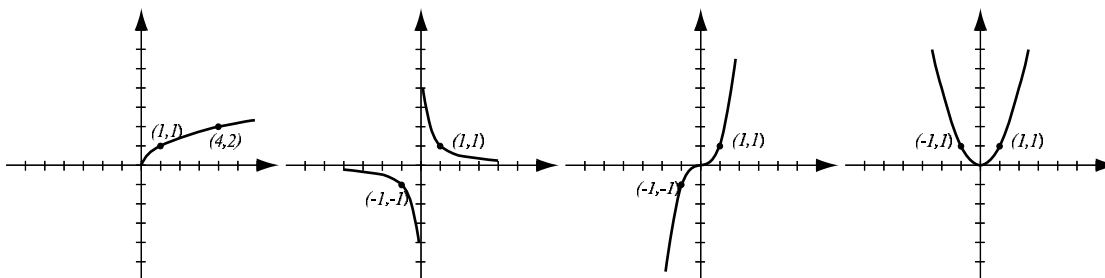
a) Find the growth rate  $r$  and write the function that describes the population of B.C.

b) If the city continues to grow at the same rate, what will be its population four years from now?

**Bonus** (10pts) What is a better deal: a savings account with a 4% interest rate, compounded monthly, or a savings account with a 4.125% interest rate, compounded quarterly? (*Hint: compare them over one year.*)

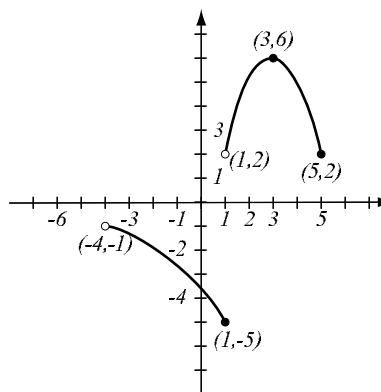


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (10pts) Use the graph of the function  $f$  at right to answer the following questions.

- Find  $f(3)$  and  $f(1)$ .
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = -4$ ?
- Find all  $x$  for which  $f(x) \geq 0$ .



3. (6pts) Simplify and write the answer so all exponents are positive:

$$\frac{(6x^{-3}y^{-4})^3}{(12x^{-5}y^9)^2} =$$

4. (7pts) Simplify, showing intermediate steps.

$$64^{\frac{2}{3}} =$$

$$\sqrt{75x^{11}y^6} =$$

5. (6pts) Solve the equation.

$$\frac{2x - 3}{x + 1} + 4 = 3 - \frac{2x - 5}{x + 1}$$

6. (7pts) Solve the inequality and write the solution using interval notation:

$$|x + 7| \geq 4$$

7. (8pts) Find the equation of the line (in the form  $y = mx + b$ ) that is perpendicular to the line  $3x + 2y = 7$  and passes through the point  $(2, 5)$ .

8. (4pts) Find the domain of the function  $f(x) = \frac{3x - 1}{\sqrt{2x - 5}}$  and write it in interval notation.

9. (24pts) Let  $f(x) = x^4 - 6x^2 - 7$  (answer with 4 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of  $f$  (on paper!). Indicate scale on the graph.

b) Determine algebraically whether  $f$  is even, odd, or neither. Justify your answer further by examining the graph.

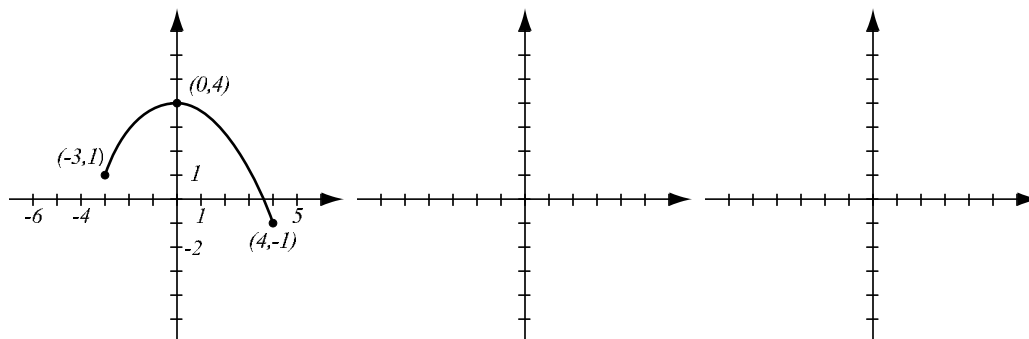
c) Algebraically find the  $x$ - and  $y$ -intercepts.

d) Find where  $f$  has a local minimum and maximum.

e) Find the intervals of increase and decrease.

f) Is the function one-to-one? Justify.

10. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $-f(x) + 3$  and  $f(\frac{1}{2}x)$  and label all the relevant points.



11. (14pts) The quadratic function  $f(x) = -x^2 - 4x + 12$  is given. Do the following without using the calculator.
- Find the  $x$ - and  $y$ -intercepts of its graph, if any.
  - Find the vertex of the graph.
  - Sketch the graph of the function.

12. (7pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log(1000\sqrt[4]{x^7y^{13}}) =$$

**13.** (7pts) Write as a single logarithm. Simplify if possible.

$$2 \log_5(x^2 - 16) - 3 \log_5(x + 4) - 2 \log_5(x - 4) =$$

**14.** (8pts) Solve the equation.

$$5^{7x+2} = 3^{4x}$$

**15.** (12pts) In 1997, the population of the island Greauf Ast was 320, in 2002, it was 414. Assume that the population grows according to the usual formula  $N(t) = N_0 e^{rt}$ .

- Find the growth rate  $r$  and the function that describes the population of Greauf Ast.
- If the island continued to grow at the same rate, what was its population in 2008?

**16.** (12pts) A runner and a walker cover the same distance. The runner finishes in half an hour, while the walker takes an hour and 15 minutes. How fast is each person going if the runner runs 4mph faster than the walker? Write down the meaning of the variable you use.

**Bonus** (10pts) Find the equation of the line that passes through the point  $(1, 2)$  and the center of the circle  $x^2 + y^2 - 6x + 8y + 1 = 0$ . Draw the line and the circle.