

1. (20pts) Let  $f(x) = \sqrt{3x + 16}$ ,  $g(x) = \frac{1}{x} + 4$ .

Find the following (simplify where possible):

$$\frac{f}{g}(2) =$$

$$(f \cdot g)(x) =$$

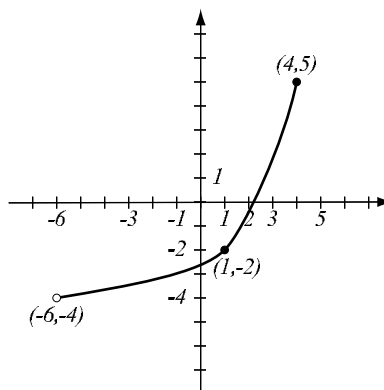
$$(g \circ f)(0)$$

$$(f \circ g)(x) =$$

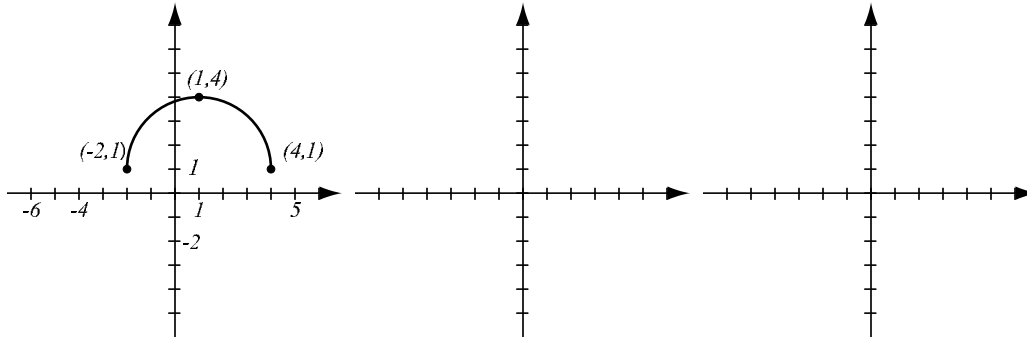
The domain of  $(f - g)(x)$

2. (7pts) The graph of a function  $f$  is given.

- a) Is this function one-to-one? Justify.  
 b) If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



3. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $f(x - 1) + 3$  and  $f(2x)$  and label all the relevant points.



4. (15pts) The quadratic function  $f(x) = -x^2 - 3x + 10$  is given. Do the following without using the calculator.
- Find the  $x$ - and  $y$ -intercepts of its graph, if any.
  - Find the vertex of the graph.
  - Sketch the graph of the function.
  - What is the range of the function?

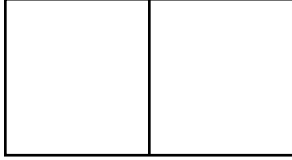
5. (22pts) Consider the polynomial  $f(x) = x^4 - 9x^3 + 18x^2$ .

- a) Find the  $y$ - and  $x$ -intercepts algebraically. What are the multiplicities of the zeroes of  $f$ ?
- b) Use your calculator to draw the graph of the function (on paper!).
- c) Find all the turning points (4 decimal points accuracy).
- d) Describe the end behavior of  $f$ .

6. (10pts) Let  $f(x) = (3x + 2)^3$ .

- a) Find the formula for  $f^{-1}$ .
- b) Show that  $(f \circ f^{-1})(y) = y$ .

7. (16pts) Farmer Tom has 5000 meters of fencing. He would like to enclose a rectangular area and divide it in half with a fence so that the area is the largest possible. Find the dimensions of the enclosure that will give the greatest area. What is the greatest area?



**Bonus.** (10pts) Find the point on the line  $y = 5 - 3x$  that is closest to the point  $(-1, 3)$ . Draw a picture. Hints: Set up the expression for the distance  $d$  between a generic point  $(x, y)$  and the point  $(-1, 3)$ . Then express  $d$  only in terms of  $x$ , and minimize  $d^2$  (you will need to simplify  $d^2$ ).