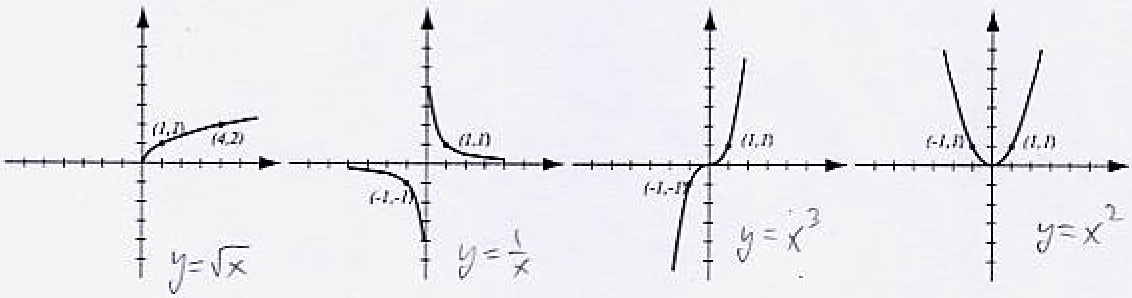
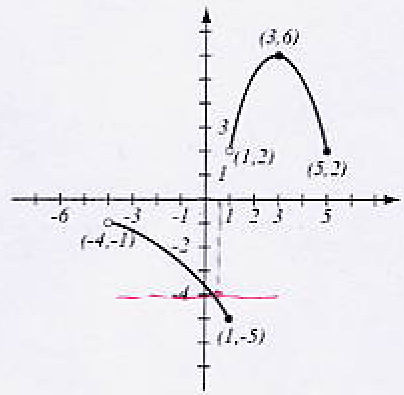
Fall '10/MAT 140/Final exam **Name:** Solution *Show all your work.*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=\sqrt{x}$ $\quad y=\frac{1}{x}$ $\quad y=x^3$ $\quad y=x^2$

2. (10pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(3)$ and $f(1)$. $f(3)=6,\ f(1)=-5$

b) What is the domain of f? $(-4,5]$

c) What is the range of f? $[-5,-1)\cup[2,6]$

d) What are the solutions of the equation $f(x)=-4$? $x=-\frac{1}{2}$

e) Find all x for which $f(x)\geq 0$.

Part above x-axis: $(1,5]$

3. (6pts) Simplify and write the answer so all exponents are positive:

$$\frac{(6x^{-3}y^{-4})^3}{(12x^{-5}y^9)^2}=\frac{6^3x^{-9}y^{-12}}{12^2x^{-10}y^{18}}=\frac{6\cdot 6\cdot 6\,x^{-9-(-10)}y^{-12-18}}{12\cdot 12}=\frac{3xy^{-30}}{2}=\frac{3x}{2y^{30}}$$

4. (7pts) Simplify, showing intermediate steps.

$$64^{\frac{2}{3}}=\left(\sqrt[3]{64}\right)^2=4^2=16$$

$$\sqrt{75x^{11}y^6}=\sqrt{3\cdot 25\cdot x^{10}\cdot x\cdot y^6}=5x^5y^3\sqrt{3x}$$

5. (6pts) Solve the equation.

$$\frac{2x-3}{x+1}+4=3-\frac{2x-5}{x+1} \quad \Big|-3$$

$$\frac{2x-3}{x+1}+1=-\frac{2x-5}{x+1} \quad \Big|\cdot(x+1)$$

$$\frac{2x-3}{\cancel{x+1}}\cdot\cancel{(x+1)}+1\cdot(x+1)=-\frac{2x-5}{\cancel{x+1}}\cdot\cancel{(x+1)}$$

$$2x-3+x+1=-2x+5$$

$$3x-2=-2x+5 \quad \Big|+2x+2$$

$$5x=7 \quad \Big|\div 5$$

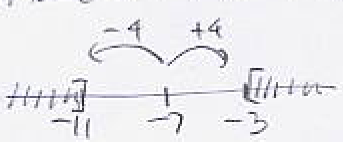
$$x=\frac{7}{5}$$ ← it does not give 0 in denominator in original equation, so is a solution

6. (7pts) Solve the inequality and write the solution using interval notation:

$|x+7| \geq 4$

$|x-(-7)| \geq 4$

distance from x to $-7 \geq 4$



$(-\infty,-11]\cup[-3,\infty)$

7. (8pts) Find the equation of the line (in the form $y = mx + b$) that is perpendicular to the line $3x + 2y = 7$ and passes through the point $(2, 5)$.

$$3x+2y=7 \quad \Big|-3x$$

$$2y=-3x+7 \quad \Big|\div 2$$

$$y=-\frac{3}{2}x+\frac{7}{2}$$

Has slope $-\frac{3}{2}$, so perpendicular line will have slope $-\frac{1}{-\frac{3}{2}}=\frac{2}{3}$

Perpendicular line:

$$y-5=\frac{2}{3}(x-2)$$

$$y=\frac{2}{3}x-\frac{4}{3}+5$$

$$y=\frac{2}{3}x+\frac{11}{3}$$

8. (4pts) Find the domain of the function $f(x) = \dfrac{3x-1}{\sqrt{2x-5}}$ and write it in interval notaition.

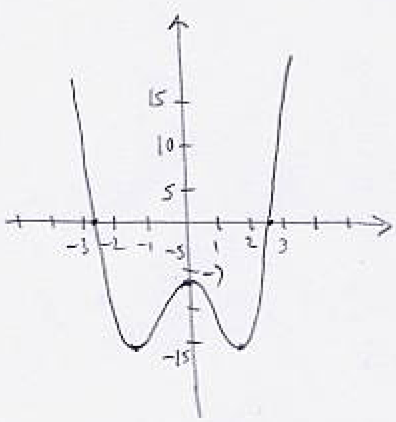
Must have $2x-5>0$ (can't have equal to 0 in denom.)

$2x>5$ Domain: $(\frac{5}{2}, \infty)$

$x>\frac{5}{2}$

9. (24pts) Let $f(x) = x^4 - 6x^2 - 7$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
c) Algebraically find the x- and y-intercepts.
d) Find where f has a local minimum and maximum.
e) Find the intervals of increase and decrease.
f) Is the function one-to-one? Justify.

a)



c) y-int: $f(0) = -7$

x-int: $x^4 - 6x^2 - 7 = 0$

$u = x^2$

$u^2 - 6u - 7 = 0$

$(u-7)(u+1) = 0$

$u = 7, -1$

$x^2 = 7$ $\quad$ $x^2 = -1$

$x = \pm\sqrt{7}$ $\quad$ no real solution

≈ 2.6458

b) $f(-x) = (-x)^4 - 6(-x)^2 - 7$

$= x^4 - 6x^2 - 7 = f(x)$

f is even - can tell from graph because it is symmetric about the y-axis.

d) f has a local min at $x = 1.7321$ whose value is $y = -16$

" " " " " $x = -1.7321$ " $y = -16$

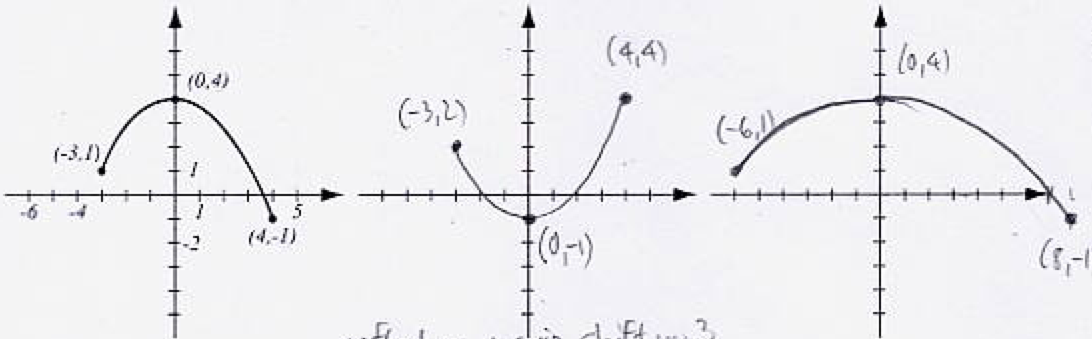
f has a local max at $x = 0$ whose value is $y = -7$

e) increasing on: $(-1.7321, 0) \cup (1.7321, \infty)$

decreasing on: $(-\infty, -1.7321) \cup (0, 1.7321)$

f) Not one-to-one, because it does not pass the horizontal line test.

10. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x)+3$ and $f(\frac{1}{2}x)$ and label all the relevant points.



stretch horizontally by factor $\frac{1}{\frac{1}{2}} = 2$

reflect in x-axis, shift up 3

11. (14pts) The quadratic function $f(x) = -x^2 - 4x + 12$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) $-x^2-4x+12=0$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6,2$

y-int: $f(0)=12$

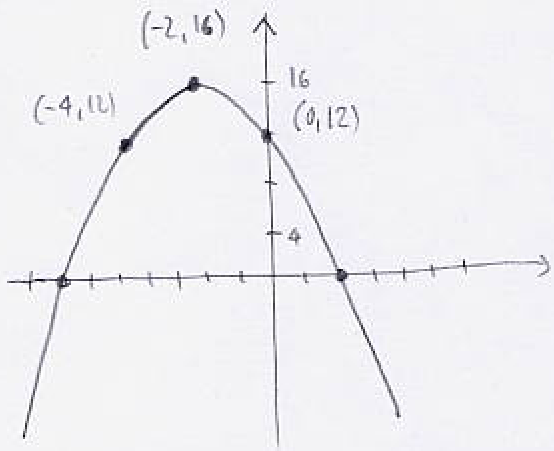
b) Vertex:

$x = -\frac{-4}{2\cdot(-1)} = -2$

$y = -(-2)^2 - 4\cdot(-2)+12$

$= -4+8+12$

$= 16$



Opens down because coefficient with x^2 is negative.

12. (7pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log(1000\underbrace{\sqrt[4]{x^7}}_{x^{\frac{7}{4}}}y^{13}) = \log 1000 + \log x^{\frac{7}{4}} + \log y^{13}$

$= 3 + \frac{7}{4}\log x + 13\log y$

13. (7pts) Write as a single logarithm. Simplify if possible.

$2\log_5(x^2-16) - 3\log_5(x+4) - 2\log_5(x-4) = \log_5(x^2-16)^2 - \log_5(x+4)^3 - \log_5(x-4)^2$

$= \log_5(x^2-16)^2 - \left(\log_5(x+4)^3 + \log_5(x-4)^2\right) = \log_5(x^2-16)^2 - \log_5\left((x+4)^3(x-4)^2\right)$

$= \log_5 \frac{(x^2-16)^2}{(x+4)^3(x-4)^2} = \log_5 \frac{((x-4)(x+4))^2}{(x+4)^3(x-4)^2} = \log_5 \frac{\cancel{(x-4)^2}(x+4)^{\cancel{2}}}{\cancel{(x+4)^3}(x-4)^{\cancel{2}}} = \log_5 \frac{1}{(x+4)}$

$= -\log_5(x+4)$

14. (8pts) Solve the equation.

$5^{7x+2} = 3^{4x} \quad | \ln$

$\ln 5^{7x+2} = \ln 3^{4x}$

$(7x+2)\ln 5 = 4x\ln 3$

$7x\ln 5 + 2\ln 5 = 4x\ln 3 \quad | -4x\ln 3 - 2\ln 5$

$7x\ln 5 - 4x\ln 3 = -2\ln 5$

$x(7\ln 5 - 4\ln 3) = -2\ln 5$

$x = \frac{-2\ln 5}{7\ln 5 - 4\ln 3} = \frac{2\ln 5}{4\ln 3 - 7\ln 5} \approx -0.4684$

15. (12pts) In 1997, the population of the island Greauf Ast was 320, in 2002, it was 414. Assume that the population grows according to the usual formula $N(t) = N_0e^{rt}$.
a) Find the growth rate r and the function that describes the population of Greauf Ast.
b) If the island continued to grow at the same rate, what was its population in 2008?

a) $N(t) = 320\, e^{rt}$

t = years since 1997

$414 = 320\, e^{r\cdot 5} \quad | \div 414$

$\frac{\cancel{414}\ 207}{\cancel{320}\ 160} = e^{5r} \quad | \ln$

$\ln \frac{207}{160} = \ln e^{5r}$

$\ln \frac{207}{160} = 5r$

$r = \frac{\ln \frac{207}{160}}{5} \approx 0.05151$

$N(t) = 320\, e^{0.05151t}$

b) $N(11) = 320\, e^{0.05151\cdot 11} \approx 563.9243$

2008 is 11 years since 1997

About 564 people

16. (12pts) A runner and a walker cover the same distance. The runner finishes in half an hour, while the walker takes an hour and 15 minutes. How fast is each person going if the runner runs 4mph faster than the walker? Write down the meaning of the variable you use.

$v+4,\ \frac{1}{2}$ hour $\qquad v =$ walker's speed

$v,\ 1$ hr 15 min

Walker walks at 2.6667 mph,
Runner runs at 6.6667 mph

$(v+4)\cdot\frac{1}{2} = v\cdot 1.25$

$0.5v + 2 = 1.25v \quad | -0.5v$

$2 = 0.75v$

$v = \frac{2}{0.75} = \frac{2}{\frac{3}{4}} = \frac{2}{1}\cdot\frac{4}{3} = \frac{8}{3} \approx 2.6667$ mph

Bonus (15pts) Find the equation of the line that passes through the point $(1, 2)$ and the center of the circle $x^2 + y^2 - 6x + 8x + 1 = 0$. Draw the line and the circle.

$x^2+y^2-6x+8y+1=0 \quad | -1$

$x^2-6x+y^2+8y=-1 \quad | +3^2+4^2$

$x^2-6x+3^2+y^2+8y+4^2=-1+9+16$

$(x-3)^2+(y+4)^2=24$

$(x-3)^2+(y-(-4))^2=24$

Center $= (3,-4)$

Radius $= \sqrt{24} = 2\sqrt{6} \approx 4.8990$

Line passes through $(1,2)$ and $(3,-4)$

$m = \frac{-4-2}{3-1} = -\frac{6}{2} = -3$

$y-2=-3(x-1)$

$y=-3x+3+2=-3x+5$