1. (8pts) Evaluate without using the calculator:

$\log_5 25 = 2$ $\qquad$ $\log_{1000} 10 = \frac{1}{3}$ $\qquad$ $\log_c \sqrt[6]{c^7} = \frac{7}{6}$ $\qquad$ $\log_{25} 125 = \frac{3}{2}$ $\quad$ (think root)

$5^? = 25$ $\qquad$ $1000^? = 10$ $\qquad$ $c^? = c^{\frac{7}{6}}$ $\qquad$ $\sqrt{25} = 5,\ 5^? = 125$

$\sqrt[3]{1000} = 10$ $\qquad\qquad\qquad$ $125 = (\sqrt{25})^3 = 5^{\frac{3}{2}}$

2. (4pts) Use your calculator to find $\log_7 3$ with accuracy 4 decimal places. Show how you obtained your number.

$\log_7 3 = \frac{\ln 3}{\ln 7} = 0.5646$

3. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log((4y+1)^5 \sqrt[3]{5x-6}) = \log(4y+1)^5 + \log(5x-6)^{\frac{1}{3}}$

$\qquad\qquad\qquad\qquad = 5\log(4y+1) + \frac{1}{3}\log(5x-6)$

$\log_8\left(\frac{\sqrt{y^3}}{64x^7}\right) = \log_8 \sqrt{y^3} - \log_8(64x^7) = \log_8 y^{\frac{3}{2}} - \left(\underbrace{\log_8 64 + \log_8 x^7}_{=2}\right)$

$\qquad\qquad\qquad = \frac{3}{2}\log_8 y - 2 - 7\log_8 x$

4. (13pts) Write as a single logarithm. Simplify if possible.

$4\log(2x^3) - 3\log(4x^5) = \log(2x^3)^4 - \log(4x^5)^3 = \log\frac{(2x^3)^4}{(4x^5)^3} = \log\frac{16x^{12}}{64x^{15}} = \log\frac{1}{4x^3}$

$\qquad\qquad\qquad\qquad\qquad\qquad = -\log(4x^3)$

$3\ln(x^2-x-30) + \ln(x+5) - 2\ln(x-6) = \ln(x^2-x-30)^3 + \ln(x+5) - \ln(x-6)^2$

$= \ln\frac{(x^2-x-30)^3(x+5)}{(x-6)^2} = \ln\frac{((x-6)(x+5))^3(x+5)}{(x-6)^2} = \ln\frac{(x-6)^3(x+5)^3(x+5)}{(x-6)^2}$

$= \ln((x-6)(x+5)^4)$

5. (8pts) How much should you invest in an account bearing 4.25%, compounded quarterly, if you wish to have $4,000 in four years?
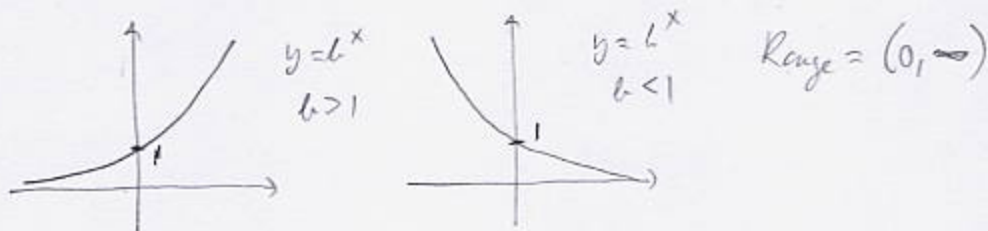
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$4000 = P\left(1 + \frac{0.0425}{4}\right)^{4\cdot4}$$

$$4000 = P\left(1.010625\right)^{16}$$
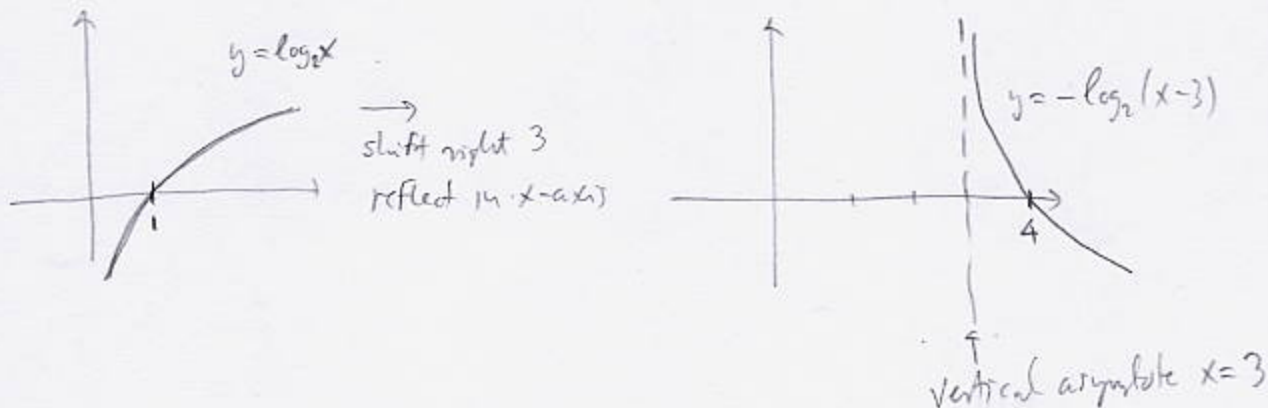
$$P = \frac{4000}{1.010625^{16}} = 3377.69$$

6. (8pts) Draw the graphs of the exponential function $f(x) = b^x$ for $0 < b < 1$ and for $b > 1$. What is the range in both cases?

$y = b^x$
$b > 1$

$y = b^x$
$b < 1$

$Range = (0, \infty)$

7. (12pts) Let $f(x) = -\log_2(x - 3)$.
a) What is the domain of f?
b) Use transformations to draw the graph of f, starting with the graph of $y = \log_2 x$. Indicate the x-intercept and any asymptotes.

a) Must have $x - 3 > 0$

$$x > 3$$

domain $(3, \infty)$

$y = \log_2 x$

shift right 3
reflect in x-axis

$y = -\log_2(x-3)$

vertical asymptote $x = 3$

Solve the equations.

8. (8pts) $3^{x+2} = \left(\dfrac{1}{9}\right)^{2-x}$

$$3^{x+2} = (3^{-2})^{2-x}$$

$$3^{x+2} = 3^{-4+2x}$$

$$x+2 = -4+2x \quad |-x+4$$

$$6 = x$$

9. (12pts) $\log_9(x+6) + \log_9(x+4) = \log_9(3x+12)$

$$\log_9\big((x+6)(x+4)\big) = \log_9(3x+12) \qquad 9^{-} \qquad x = -3, -4 \qquad \boxed{x = -3} \text{ is the solution}$$

Test back:

$$(x+6)(x+4) = 3x+12 \qquad\qquad \log_9(-4+6) + \log_9(\underbrace{-4+0}_{6}) = \log_9(-12+12$$

$$x^2 + 10x + 24 = 3x + 12 \qquad\qquad \text{not defined}$$

$$x^2 + 7x + 12 = 0 \qquad\qquad \log_9(-3+6) + \log_9(-3+4) = \log_9(-9+12)$$

$$(x+3)(x+4) = 0 \qquad\qquad \log_9 3 + \underbrace{\log_9 1}_{=0} = \log_9 3 \quad ok$$

14

11. (~~12~~pts) Five years ago, the population of Bunny City was 45,000, today, it is 57,000. Assume that the population grows according to the usual formula $N(t) = N_0 e^{rt}$.
a) Find the growth rate r and write the function that describes the population of B.C.
b) If the city continues to grow at the same rate, what will be its population four years from now?

a) $N = 45 e^{rt}$ (in thousands)

$57 = 45 e^{r \cdot 5}$ | $\div 45$

$\frac{57}{45} = e^{5r}$ | $\ln$

$\ln \frac{57}{45} = \ln e^{5r}$

$\ln \frac{57}{45} = 5r$

$r = \frac{\ln \frac{57}{45}}{5} = 0.04728$

$N(t) = 45 e^{0.04728t}$

b) 4 years from now is 9 years since beginning.

$N(9) = 45 e^{0.04728 \cdot 9}$

$= 68.8660$

About 68,866 inhabitants.

Bonus (10pts) What is a better deal: a savings account with a 4% interest rate, compounded monthly, or a savings account with a 4.125% interest rate, compounded quarterly? *(Hint: compare them over one year.)*

Consider what happens to $1 invested:

At 4%: $A = 1 \cdot \left(1 + \frac{0.04}{12}\right)^{12 \cdot 1} = \left(1 + 0.00333...\right)^{12} = 1.040741$ (APY = 4.0741%)

At 4.125%: $A = 1 \cdot \left(1 + \frac{0.04125}{4}\right)^{4 \cdot 1} = \left(1.0103125\right)^4 = 1.041892$ ← slightly better (APY = 4.1892%)

4.125% comp. quarterly is a better deal.