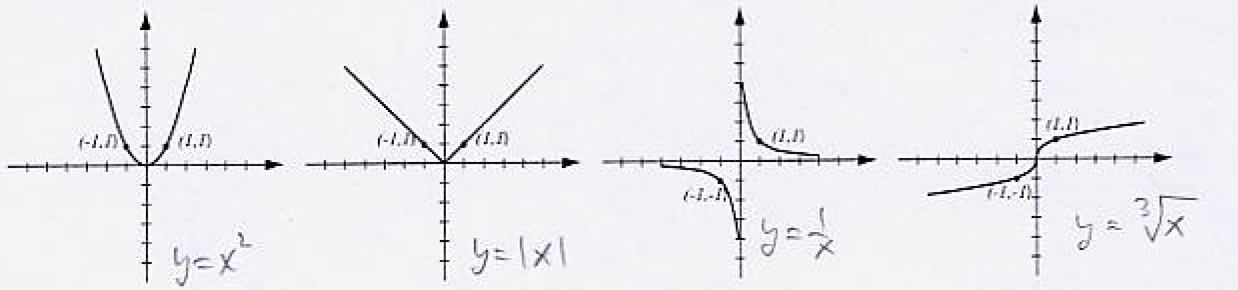
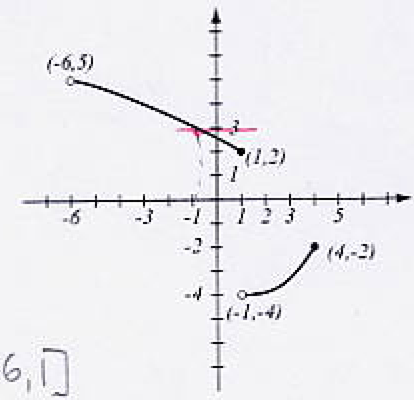
Fall '10/MAT 140/Exam 3 Name: Solution *Show all your work.*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=x^2$ $y=|x|$ $y=\frac{1}{x}$ $y=\sqrt[3]{x}$

2. (10pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(4)$ and $f(1)$. $f(4)=-2$ $f(1)=2$

b) What is the domain of f? $(-6,4]$

c) What is the range of f? $(-4,-2]\cup[2,5)$

d) What are the solutions of the equation $f(x)=3$? $x=-1$

e) Find all x for which $f(x)\geq 2$. x in $(-6,1]$

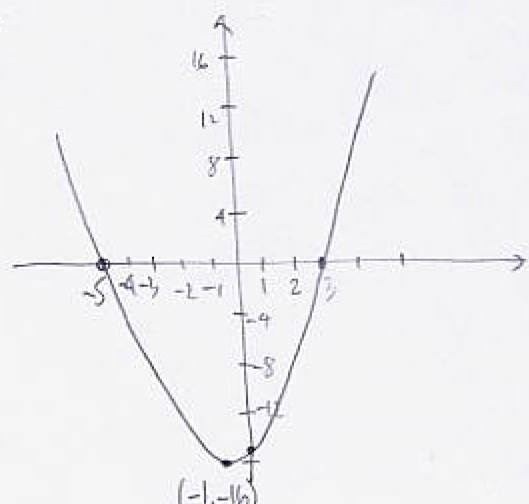
3. (15pts) The quadratic function $f(x)=x^2+2x-15$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) Is the function one-to-one? Justify.



a) y-int: $f(0)=-15$

x-int: $x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5,3$

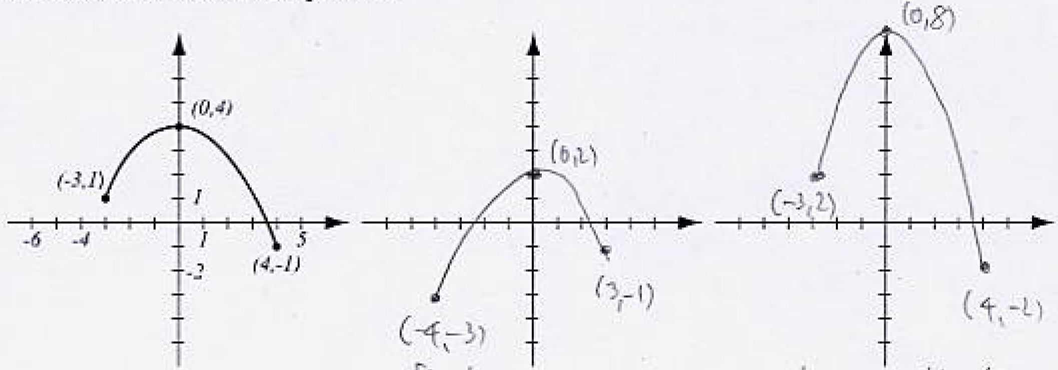
b) $h=-\frac{2}{2\cdot 1}=-1$

$k=f(-1)=(-1)^2+2\cdot(-1)-15$

$=-16$

d) f is not one-to-one - it fails the horizontal line test

4. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)-2$ and $2f(x)$ and label all the relevant points.



reflect over y-axis, down 2

stretch vertically, factor = 2

5. (18pts) Let $f(x) = \dfrac{x-3}{x^2+3x \cancel{+6} -4}$, $g(x) = x+2$.

Find the following (simplify where possible):

$\dfrac{f}{g}(2) = \dfrac{f(2)}{g(2)} = \dfrac{\frac{-1}{6}}{4} = -\dfrac{1}{6}\cdot\dfrac{1}{4}$

$= -\dfrac{1}{24}$

$(f\cdot g)(x) = \dfrac{x-3}{x^2+3x-4}\cdot(x+2) = \dfrac{(x-3)(x+2)}{(x+4)(x-1)}$

$(g\circ f)(0) = g(f(0)) = g\left(\dfrac{-3}{-4}\right)$

$= \dfrac{3}{4} + 2 = \dfrac{11}{4}$

$(f\circ g)(x) = f(g(x)) = f(x+2)$

$= \dfrac{(x+2)-3}{(x+2)^2+3(x+2)-4} = \dfrac{x-1}{x^2+4x+4+3x+6-4}$

$= \dfrac{x-1}{x^2+7x+6} = \dfrac{x-1}{(x+6)(x+1)}$

The domain of $(f-g)(x)$

$=$ domain of f $\cap$ $\underbrace{\text{domain of } g}_{\text{all reals}}$ $= \{x \mid x \neq -4, 1\}$

$\{x \mid x \neq -4, 1\}$

$= (-\infty,-4)\cup(-4,1)\cup(1,\infty)$

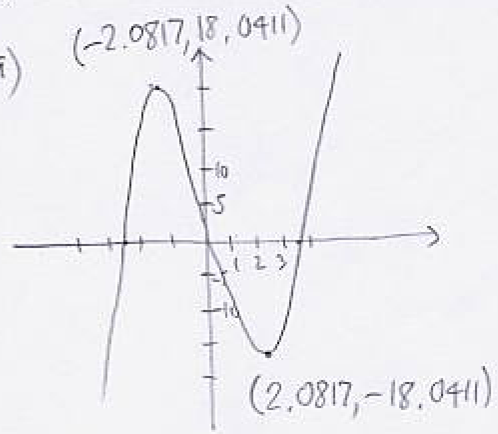
$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4, 1$

6. (21pts) Let $f(x) = x^3 - 12x$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
c) Algebraically find the x- and y-intercepts.
d) Find where f has a local minimum and maximum.
e) Find the intervals of increase and decrease.

a)



b) $f(-x) = (-x)^3 - 13(-x)$
$= -x^3 + 13x = -(x^3 - 13x)$
$= -f(x)$

f is odd

Graph of f is symmetric about the origin.

c) y-int: $f(0) = 0$
x-int: $x^3 - 13x = 0$
$x(x^2 - 13) = 0$
$x = 0$ or $x^2 - 13 = 0$
$x = \pm\sqrt{13} \approx 3.6056$

d) f has a local min at $x \approx 2.0817$ with value $y = -18.0411$
f has a local max at $x \approx -2.0817$ with value $y = 18.0411$

e) f is increasing on $(-\infty, -2.0817) \cup (2.0817, \infty)$
decreasing on $(-2.0817, 2.0817)$

7. (10pts) Let $f(x) = x^2 + 3$, $x \geq 0$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = x^2 + 3 \quad | -3$
$y - 3 = x^2 \quad | \sqrt{\ }$
$\sqrt{x^2} = \sqrt{y-3}$
$x = \sqrt{y-3}$
$f^{-1}(y) = \sqrt{y-3}$

$\sqrt{x^2} = x$ since $x \geq 0$

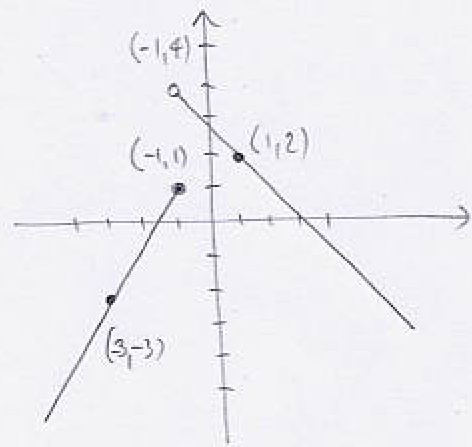
Range of f = domain of f^{-1} ← must have $y - 3 \geq 0$, $y \geq 3$
$= [3, \infty)$

8. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x+3, & \text{if } x \le -1 \\ -x+3, & \text{if } -1 < x. \end{cases}$$

x	$2x+3$
-1	1
-3	-3

x	$-x+3$
-1	4
1	2



Bonus. (10pts) Eric has 50ft of fencing (that is the length) that he will use to enclose a rectangular pen for his dog along a wall of his house (there is no fence along the wall). Follow the steps below to find the dimensions of the pen that has the greatest area.
a) Write the area of the pen in terms of x and y. Then use the condition above to help you write the area $A(x)$ as a function only of x.
b) You should have gotten a quadratic function for $A(x)$. Graph it and determine algebraically where it achieves a maximum.
c) What are the dimensions of the pen with the greatest area?

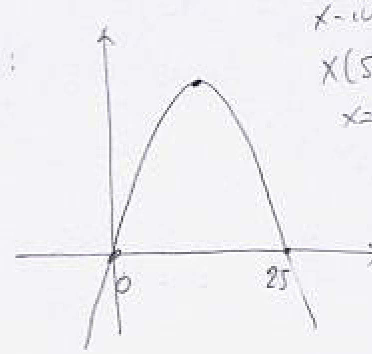
$2x+y=50$

$y=50-2x$

$A=xy=x(50-2x)$

$=-2x^2+50x$

Graph of A:



x-int:
$x(50-2x)=0$
$x=0$ or
$50-2x=0$
$x=25$

Vertex is at

$h=-\frac{50}{2\cdot(-2)}=12.5$

$k=-2\cdot 12.5^2+50\cdot 12.5$

$=312.5$ sq Ft.

Rectangle with max area is $x=12.5$, $y=25$, max area $=312.5$ sq Ft