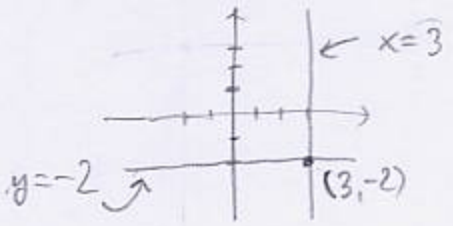
Fall '10/MAT 140/Exam 2 **Name:** Solution *Show all your work.*

1. (6pts) Write the equations of the vertical and horizontal lines that pass through the point $(3, -2)$. Draw a picture.



2. (6pts) Are the lines $2x - 3y = 5$ and $y = -\frac{2}{3}x + 6$ parallel? Explain.

$2x-3y=5 \quad |-5+3y$

$2x-5=3y \quad |\div 3$

$y = \frac{2}{3}x - \frac{5}{3}$

First line has slope $\frac{2}{3}$, second $-\frac{2}{3}$.

Slopes are not equal, so lines are not parallel.

3. (5pts) Solve the equation.

$|2x + 5| = 11$

$2x+5=11$ or $2x+5=-11$

$2x=6 \qquad 2x=-16$

$x=3$ or $x=-8$

4. (14pts) Solve the inequalities and write the solution using interval notation:

$4 - 3x > 7 \quad |-4$

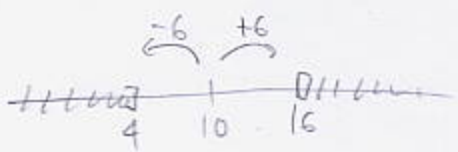
$-3x > 3 \quad |\div -3$

$x < -1$

$(-\infty, -1)$

$|3x - 10| \geq 6$

distance from $3x$ to $10 \geq 6$



$3x \leq 4$ or $3x \geq 16$

$x \leq \frac{4}{3}$ or $x \geq \frac{16}{3}$

$(-\infty, \frac{4}{3}] \cup [\frac{16}{3}, \infty)$

7. (15pts) Solve the equations.

$2x^2 - x + 3 = 5 - 4x^2$ $\quad |+4x^2-5$

$6x^2 - x - 2 = 0$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 6 \cdot (-2)}}{2 \cdot 6}$

$= \dfrac{1 \pm \sqrt{49}}{12} = \dfrac{1 \pm 7}{12}$

$= \dfrac{8}{12}, -\dfrac{6}{12} = \dfrac{2}{3}, -\dfrac{1}{2}$

$x - 3 = \sqrt{33 - 8x}$ $\quad |^2$

$x^2 - 6x + 9 = 33 - 8x$ $\quad |+8x-33$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6$ or 4

Check: $-6 - 3 \stackrel{?}{=} \sqrt{33+48}$ $\quad 4-3 \stackrel{?}{=} \sqrt{33-32}$

$-9 \stackrel{?}{=} \sqrt{81}$ no $\quad 1 = \sqrt{1}$ yes

only 4 is the solution

8. (10pts) Below is an equation of a circle. Find the center and radius of the circle and draw the circle.

$x^2 + y^2 + 8x + 12y - 12 = 0$ $\quad |+4^2+6^2$

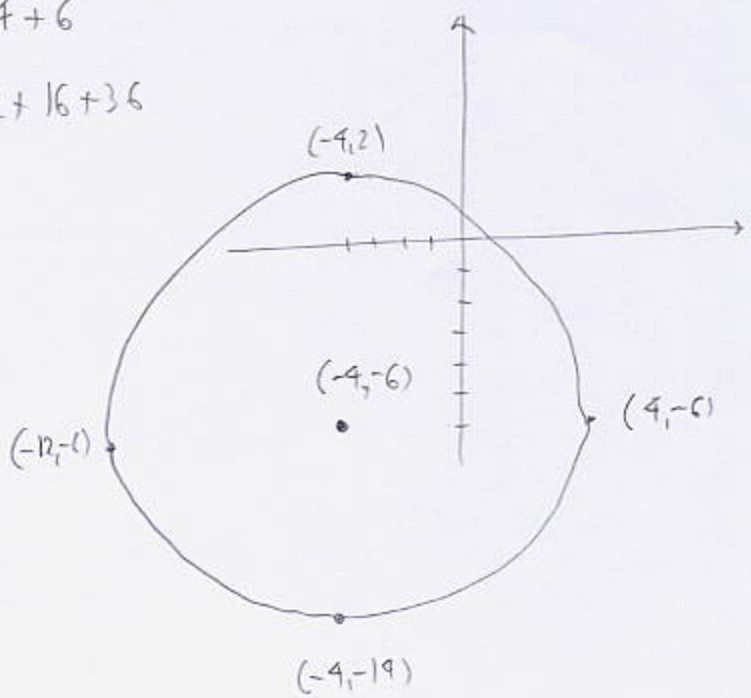
$x^2 + 8x + 4^2 + y^2 + 12y + 6^2 = 12 + 16 + 36$

$(x+4)^2 + (y+6)^2 = 64$

$(x-(-4))^2 + (y-(-6))^2 = 64$

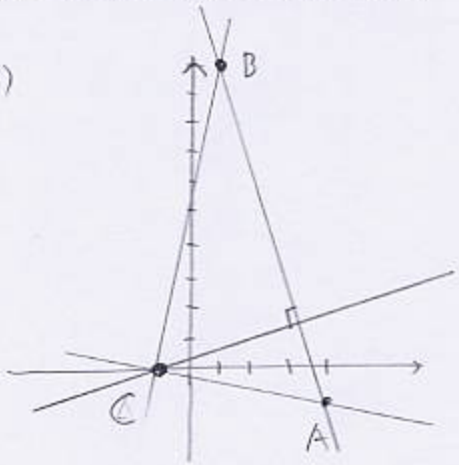
Center: $(-4, -6)$

radius: 8



5. (20pts) Let $A = (4, -1)$, $B = (1, 10)$ and $C = (-1, 0)$.
a) Draw these points in the coordinate system.
b) Show that the triangle ABC is a right triangle.
c) Find the equation of the line that passes through C and is perpendicular to the side AB.
d) Draw the line in the picture.

a)



$d(A,B) = \sqrt{(1-4)^2 + (10-(-1))^2} = \sqrt{9+121} = \sqrt{130}$

$d(B,C) = \sqrt{(-1-1)^2 + (0-10)^2} = \sqrt{4+100} = \sqrt{104}$

$d(A,C) = \sqrt{(-1-4)^2 + (0-(-1))^2} = \sqrt{25+1} = \sqrt{26}$

$\sqrt{26}^2 + \sqrt{104}^2 \stackrel{?}{=} \sqrt{130}^2$

$26 + 104 = 130$

yes, so it is a right triangle

c) slope of AB is:

$\frac{10-(-1)}{1-4} = \frac{11}{-3} = -\frac{11}{3}$

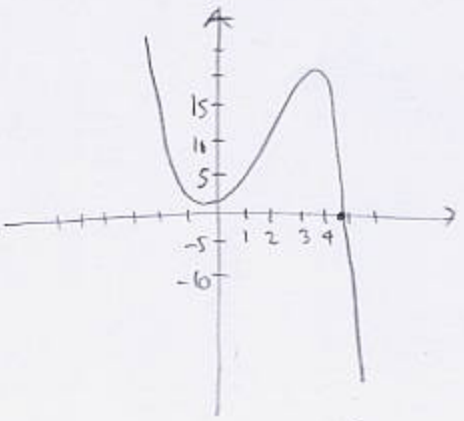
Perpendicular line has slope

$-\frac{1}{-\frac{11}{3}} = \frac{3}{11}$

Equation: $y - 0 = \frac{3}{11}(x-(-1))$

$y = \frac{3}{11}x + \frac{3}{11}$

6. (10pts) Use your calculator to accurately sketch the graph of $y = -x^3 + 3x^2 + 7x + 4$. Draw the graph here, and indicate the viewing window. Find all the $x-$ and $y-$intercepts (accuracy: 4 decimal points).

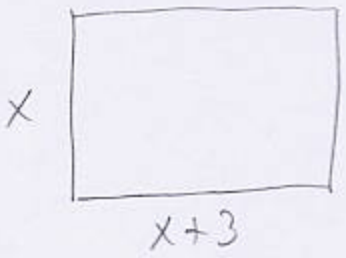


y-int: $x=0,\ y=4$

x-int: 4.6788

Window: $[-10,10] \times [-30,30]$

9. (14pts) Wilma has a rectangular picture whose area is 15 square feet. What are the dimensions of the picture if the width is 3 feet more than the height?



x = height

$x(x+3) = 15$

$x^2+3x-15=0$

$x = \frac{-3 \pm \sqrt{3^2-4\cdot 1\cdot(-15)}}{2}$

$= \frac{-3 \pm \sqrt{69}}{2}$

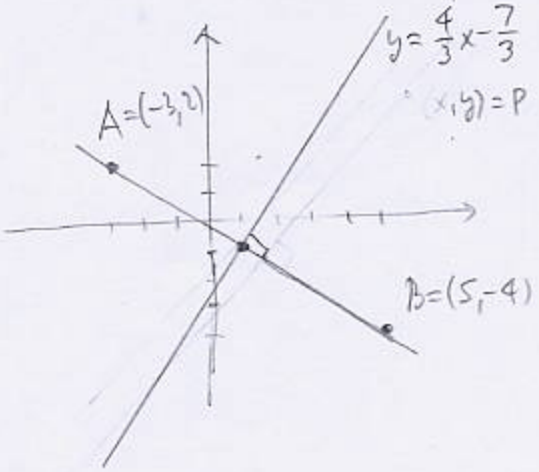
$\frac{-3-\sqrt{69}}{2} < 0$ so not a solution

$x = \frac{-3+\sqrt{69}}{2}$ height

$x+3 = \frac{-3+\sqrt{69}}{2} + 3$

$= \frac{3+\sqrt{69}}{2}$ width

Bonus (10pts) Find all points in the plane that have equal distance to $(-3, 2)$ and $(5, -4)$. Draw the picture. You should get a line perpendicular to the line between the two points. *(Hint: Set up an equation involving the distance between a generic point (x, y) and the two points. Then simplify the equation until you get the equation of a line.)*



$d(A,P) = d(B,P)$

$\sqrt{(x-(-3))^2+(y-2)^2} = \sqrt{(x-5)^2+(y-(-4))^2}$ $|^2$

$(x+3)^2+(y-2)^2 = (x-5)^2+(y+4)^2$

$x^2+6x+9+y^2-4y+4 = x^2-10x+25+y^2+8y+16$ $|-x^2 -y^2$

$6x-4y+13 = -10x+8y+41$ $|+6x-8y-13$

$16x-12y = -16x+28$

$y = \frac{-16x}{-12} + \frac{28}{-12} = \frac{4}{3}x - \frac{7}{3}$