1. (4pts) Convert to scientific notation or a decimal number:

$0.0003458 = 3.458 \times 10^{-4}$ $4.121 \times 10^7 = 41,210,000$

2. (14pts) Use formulas to expand:

$(2x - 3)(2x + 3) = (2x)^2 - 3^2$

$$= 4x^2 - 9$$

$(a^2 + 3b)^2 = (a^2)^2 + 2 \cdot a^2 \cdot 3b + (3b)^2$

$$= a^4 + 6a^2b + 9b^2$$

$(x - 4)^3 = x^3 - 3x^2 \cdot 4 + 3 \cdot x \cdot 4^2 - 4^3$

$$= x^3 - 12x^2 + 48x - 64$$

3. (14pts) Simplify, showing intermediate steps.

$\sqrt{54} = \sqrt{6 \cdot 9} = 3\sqrt{6}$ $\sqrt[5]{64} = \sqrt[5]{32 \cdot 2} = 2\sqrt[5]{2}$

$27^{-\frac{4}{3}} = \dfrac{1}{27^{\frac{4}{3}}} = \dfrac{1}{(\sqrt[3]{27})^4} = \dfrac{1}{3^4} = \dfrac{1}{81}$ $\sqrt{48x^{12}y^5} = \sqrt{16 \cdot 3 \cdot x^{12} \cdot y^4 \cdot y}$

$$= 4x^6 y^2 \sqrt{3y}$$

4. (8pts) Simplify.

$$\frac{3}{x^2+2x-15} - \frac{2}{x^2+7x+10} = \frac{3}{(x+5)(x-3)} - \frac{2}{(x+5)(x+2)}$$

$$= \frac{3(x+2) - 2(x-3)}{(x+5)(x-3)(x+2)} = \frac{3x+6-2x+6}{(x+5)(x-3)(x+2)} = \frac{x+12}{(x+5)(x-3)(x+2)}$$

5. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{(8x^6y^{-2})^{\frac{2}{3}}}{(6x^{-3}y^{\frac{5}{3}})^2} = \frac{8^{\frac{2}{3}} x^{\frac{12}{3}} y^{-\frac{4}{3}}}{36\, x^{-6}\, y^{\frac{10}{3}}} = \frac{(\sqrt[3]{8})^2 x^{4-(-6)} y^{-\frac{4}{3}-\frac{10}{3}}}{36} = \frac{\overset{1}{\cancel{4}} x^{10} y^{-\frac{14}{3}}}{\underset{9}{\cancel{36}}} = \frac{x^{10}}{9y^{\frac{14}{3}}}$$

6. (6pts) Rationalize the denominator.

$$\frac{1+4\sqrt{3}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}+8\sqrt{3}+4\sqrt{3}^2}{2^2-\sqrt{3}^2} = \frac{14+9\sqrt{3}}{4-3} = 14+9\sqrt{3}$$

7. (11pts) Simplify, with answer in form $a+bi$.

$$(3+2i)(4-9i) + 4i(5-i) = 12-27i+8i-18i^2+20i-4i^2$$

$$= 12+i+18+4 = 34+i$$

(explain) $i^{921} = (i^4)^{230} \cdot i = i$

$$\underbrace{\phantom{(i^4)^{230}}}_{1}$$

$$\frac{921}{4} = 230\frac{1}{4}$$

8. (12pts) Solve the equations.

$$\frac{3x+2}{6} + 4 = -3x + 1 - \frac{2x-1}{4} \qquad | \cdot \text{LCD}=12$$

$$\frac{3x+2}{6} \cdot \overset{2}{12} + 48 = -36x + 12 - \frac{2x-1}{4} \cdot \overset{3}{12}$$

$$6x + 4 + 48 = -36x + 12 - 6x + 3$$

$$6x + 52 = -42x + 15 \qquad \begin{vmatrix} +42x \\ -52 \end{vmatrix}$$

$$48x = -37$$

$$x = -\frac{37}{48}$$

$$\frac{5x+3}{x+1} - 7 = \frac{x}{x+1} \qquad | \cdot (x+1)$$

$$\frac{5x+3}{x+1} \cdot (x+1) - 7(x+1) = \frac{x}{x+1} \cdot (x+1)$$

$$5x + 3 - 7x - 7 = x$$

$$-2x - 4 = x \qquad | +2x$$

$$-4 = 3x$$

$$x = -\frac{4}{3} \text{ does not give a 0}$$
$$\text{in the denominators}$$
$$\text{So is a solution}$$

9. (12pts) How many liters of a 15% solution of hydrochloric acid needs to be added to 100 liters of a 35% solution of hydrochloric acid in order to get a 20% solution? Write down the meaning of the variable you use.

$$x = \text{liters of } 15\% \text{ solution}$$

$$\begin{bmatrix} 15\% \\ x \end{bmatrix} + \begin{bmatrix} 35\% \\ 100\ell \end{bmatrix} = \begin{bmatrix} 20\% \\ x+100 \end{bmatrix}$$

$$0.15x + 0.35 \cdot 100 = 0.2(x+100)$$

$$0.15x + 35 = 0.2x + 20 \qquad \begin{vmatrix} -0.15x \\ -20 \end{vmatrix}$$

$$15 = 0.05x$$

$$x = \frac{15}{0.05} = 300 \text{ liters}$$

10. (12pts) Fred, who is from Seattle, went to school in Florida. On the way to school, he took a southern route, and on his return after graduation, he took a northern route. On both trips he averaged the same speed. If the southern trek took 52 hours, the northern 60 hours, and the northern trek was 448 miles longer, how long was each trip? Write down the meaning of the variable you use.

60 hrs ├─── $s + 448$ ───┤ northern route

52 hrs ├──────┤ southern route
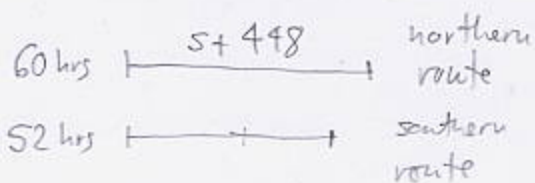
$s =$ length of southern route

Speed $v = \frac{s}{t}$ is same on both trips.

$$\frac{s+448}{60} = \frac{s}{52}$$

$$52(s+448) = 60s$$

$$52s + 23296 = 60s \quad |-52s$$

$$23296 = 8s$$

$$s = \frac{23296}{8} = 2912 \text{ mi}$$

$s = 2912$ mi southern route

$s + 448 = 3360$ mi northern route.

Bonus (10pts) Solve the equation for a complex number z. Your answer must be in form $a + bi$. (Hint: Don't fear this linear equation. Just do what you would ordinarily do, only with complex numbers.)

$$\frac{iz+1}{3-2i} + 4 = 5z + 3i \quad \Big| \cdot (3-2i)$$

$$iz + 1 + 4(3-2i) = (5z+3i)(3-2i)$$

$$iz + 1 + 12 - 8i = 15z - 10iz + 9i - 6i^2$$

$$iz + 13 - 8i = 15z - 10iz + 9i + 6 \quad \Big|\begin{array}{l} -15z + 10iz \\ -12 + 8i \end{array}$$

$$11iz - 15z + 13 - 8i = -6 + 9i \quad | -13 + 8i$$

$$z(-15 + 11i) = -7 + 17i \quad | \div (-15 + 11i)$$

$$z = \frac{-7+17i}{-15+11i} \cdot \frac{-1}{-1} = \frac{7-17i}{15-11i} \cdot \frac{15+11i}{15+11i}$$

$$= \frac{105 + 77i - 255i - 187i^2}{15^2 - (11i)^2}$$

$$= \frac{292 + 178i}{225 + 121} = \frac{2(146 - 89i)}{346}$$

$$= \frac{146 - 89i}{173} = \frac{146}{173} - \frac{89}{173}i$$