1. (7pts) The quadratic function $f(x) = -x^2 - x + 6$ is given. Do the following without using the calculator.
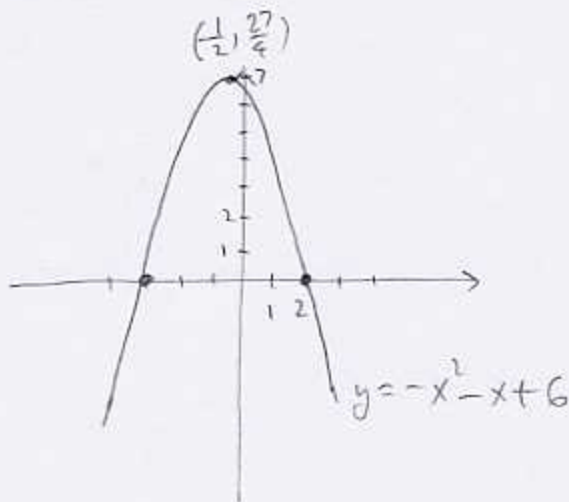a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

$\left(-\frac{1}{2}, \frac{27}{4}\right)$

$y = -x^2 - x + 6$

a) $-x^2 - x + 6 = 0$

$\quad -(x^2 + x - 6) = 0$

$\quad -(x+3)(x-2) = 0$

$\quad\quad x = -3, 2$

b) $h = -\frac{b}{2a} = -\frac{-1}{2 \cdot (-1)} = -\frac{1}{2}$

$k = \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 6 = \frac{1}{4} + \frac{1}{2} + 6 = \frac{1 + 2 + 24}{4} = \frac{27}{4} = 6\frac{3}{4}$
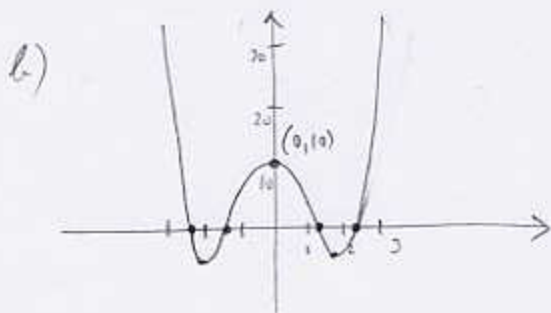
2. (13pts) Consider the polynomial $f(x) = x^4 - 7x^2 + 10$.
a) Find the y- and x-intercepts algebraically. (Recall skills from section 1.4.)
b) Use your calculator to draw the graph of the function (on paper!).
c) Find all the turning points (4 decimal points accuracy).
d) Does the polynomial have the maximal possible number of x-intercepts for its degree? How about the maximal possible number of turning points?
e) What is the range of f?

a) $x^4 - 7x^2 + 10 = 0$

$u = x^2 \quad u^2 - 7u + 10 = 0$

$\quad (u - 5)(u - 2) = 0$

$\quad u = 5, 2$

$\quad x^2 = 5 \quad x^2 = 2$

$\quad x = \pm\sqrt{5} \quad x = \pm\sqrt{2}$

$\quad = \pm 2.2361 \quad = \pm 1.4142$

$y\text{-int:}\ f(0) = 10$

b)

$(0, 10)$

c) Turning pts:
$(-1.8708, -2.25)$
$(0, 10)$
$(1.8708, -2.25)$
(note symmetry about y-axis)

d) yes, it has $4 = \deg f$
yes, it has $3 = \deg f - 1$

e) Range $= [-2.25, \infty)$
(from picture)

3. (10pts) A rancher has 30,000 linear feet of fencing and wants to enclose a rectangular field and divide it into four equal pastures with three fences parallel to one of the sides of the field (see picture in problem 62, pg. 339, and draw it here).
a) Write the area as a function of the length of one of the sides of the rectangle.
b) Sketch the graph of this function.
c) What dimensions of the rectangle give the largest area? What is the largest area?
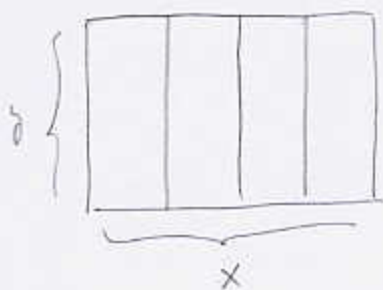
a) $2x + 5y = 30000$ $\qquad$ $A = xy = x\left(6000 - \frac{2}{5}x\right)$

$5y = 30000 - 2x$ $\qquad\qquad$ $= -\frac{2}{5}x^2 + 6000x$

$y = 6000 - \frac{2}{5}x$ $\qquad\qquad$ quadratic function

b) Intercepts: $x\left(6000 - \frac{2}{5}x\right) = 0$

$x = 0$ or $\quad 6000 - \frac{2}{5}x = 0$

$\qquad\qquad 6000 = \frac{2}{5}x \ \Big|\cdot\frac{5}{2}$

$\qquad\qquad 15000 = x$

$\qquad\qquad x \geqslant 0$

$A = -\frac{2}{5}x^2 + 6000x$
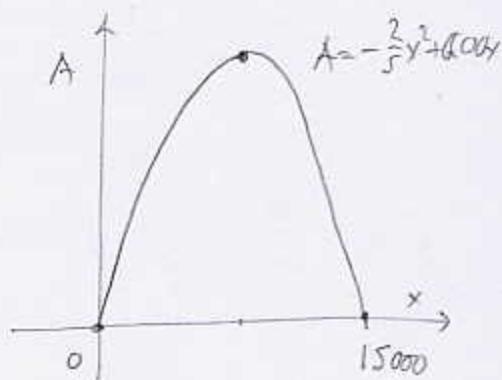
c) vertex is at $h = 7500 = -\frac{6000}{2\cdot(-\frac{2}{5})}$

$\qquad k = 7500 \cdot \left(6000 - \frac{2}{5}\cdot 7500\right)$

$\qquad = 7500 \cdot 3000$

$\qquad = 22,500,000$

Dimensions of rectangle with biggest area

are $\quad 7500 \times 3000$

$\left(y = 6000 - \frac{2}{5}\cdot 7500 = 3000\right)$