Fall '09/MAT 140/Worksheet 7 **Name:** Solution *Show all your work.*

1. (9pts) Let $f(x) = x^2 + 5x - 7$, $g(x) = 3x + 2$. Find the following functions (simplify where possible):

$(f+g)(x) = f(x) + g(x)$
$= x^2 + 5x - 7 + 3x + 2$
$= x^2 + 8x - 5$

$\frac{f}{g}(3) = \frac{f(3)}{g(3)} = \frac{3^2 + 5\cdot 3 - 7}{3\cdot 3 + 2} = \frac{17}{11}$

$(f \circ g)(0) = f(g(0))$
$= f(2) = 2^2 + 5\cdot 2 - 7$
$= 7$

$(f \circ g)(x) = f(g(x)) = f(3x+2)$
$= (3x+2)^2 + 5(3x+2) - 7$
$= 9x^2 + 12x + 4 + 15x + 10 - 7$
$= 9x^2 + 27x + 7$

$(g \circ g)(x) = g(g(x)) = g(3x+2)$
$= 3(3x+2) + 2$
$= 9x + 6 + 2$
$= 9x + 8$

$(g \circ f)(x) = g(f(x)) = g(x^2 + 5x - 7)$
$= 3(x^2 + 5x - 7) + 2$
$= 3x^2 + 15x - 19$

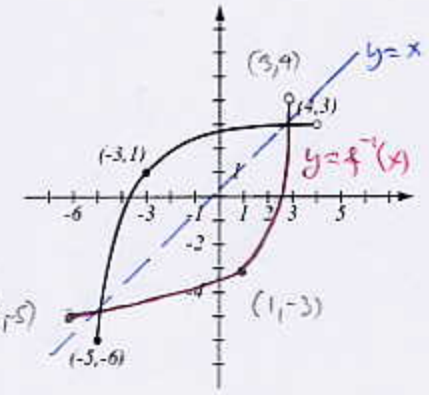
2. (5pts) Consider the function $h(x) = \frac{4}{x + \sqrt{x}}$. Find functions f and g so that $h(x) = (f \circ g)(x)$. Find two different solutions to this problem, neither of which is the "stupid" one.

$h(x) = \frac{4}{x+\sqrt{x}} = 4 \cdot \frac{1}{x+\sqrt{x}}$

Some examples:

$g(x) = x + \sqrt{x}$	$g(x) = \frac{1}{x+\sqrt{x}}$	$g(x) = \sqrt{x}$	$g(x) = x + \sqrt{x} - 1$
$f(x) = \frac{4}{x}$	$f(x) = 4 \cdot x$	$f(x) = \frac{4}{x^2 + x}$	$f(x) = \frac{4}{x+1}$

3. (5pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) yes, it passes the horizontal line test

b) swap coordinates on pts, reflect graph in line $y=x$ (blue line)

4. (6pts) Let $g(x) = \dfrac{2x}{x-7}$. Find the formula for g^{-1} and find the domain and range of g.

solve for x: $y = \dfrac{2x}{x-7} \quad | \cdot (x-7)$

$y(x-7) = 2x$

$yx - 7y = 2x$

$yx - 2x = 7y$

$x(y-2) = 7y \quad | \div (y-2)$

$x = \dfrac{7y}{y-2} \qquad g^{-1}(y) = \dfrac{7y}{y-2}$

Domain of g: $x-7=0$, $x=7$

$= \{x \mid x \neq 7\} = (-\infty, 7) \cup (7, \infty)$

Range of g = domain of g^{-1} =

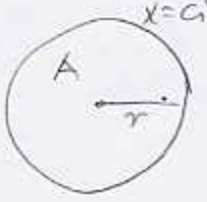
$= \{y \mid y \neq 2\} = (-\infty, 2) \cup (2, \infty)$

$y-2=0$

$y=2$

5. (5pts) Suppose you are fencing in a circular area for your dog. Let x be the length of fencing you bought.
a) Express the radius of the circle r as a function of x.
b) Express the area of the enclosure first as a function of r, then as a function of x.

a) x = circumference



$2\pi r = x$

$r = \dfrac{x}{2\pi}$

b) $A = \pi r^2 = \pi \left(\dfrac{x}{2\pi}\right)^2$

$= \pi \dfrac{x^2}{4\pi^2_1} = \dfrac{x^2}{4\pi}$