Fall '09/MAT 140/Worksheet 2 **Name:** Solution *Show all your work.*

1. (8pts) Simplify.

$$\frac{2x-3}{x^2+2x-24} - \frac{x-7}{x^2+4x-12} = \frac{2x-3}{(x+6)(x-4)} - \frac{x-7}{(x+6)(x-2)} = \frac{(2x-3)(x-2)-(x-7)(x-4)}{(x+6)(x-4)(x-2)}$$

prod $=-24$, sum $=2$: $6,-4$ prod $=-12$, sum $=4$: $6,-2$

$$= \frac{2x^2-7x+6-(x^2-11x+28)}{(x+6)(x-4)(x-2)} = \frac{2x^2-x^2-7x+11x+6-28}{(x+6)(x-4)(x-2)} = \frac{x^2+4x-22}{(x+6)(x-4)(x-2)}$$

does not factor

prod $=-22$, sum $=4$: $22,-1$ ($-22,1$ no) $2,-11$ ($-2,11$ no)

$$\frac{\frac{3x}{x+4}-1}{2+\frac{16x-8}{x^2-16}} = \frac{\frac{3x-(x+4)}{x+4}}{\frac{2(x^2-16)+16x-8}{x^2-16}} = \frac{2x-4}{x+4} \cdot \frac{x^2-16}{2x^2+16x-40}$$

$$= \frac{2(x-2)}{x+4} \cdot \frac{(x-4)(x+4)}{2(x^2+8x-20)} = \frac{(x-2)(x-4)}{(x-2)(x+10)} = \frac{x-4}{x+10}$$

prod $=-20$, sum $=8$: $10,-2$

2. (6pts) Simplify, showing intermediate steps.

$\sqrt{360} = \sqrt{36 \cdot 10} = 6\sqrt{10}$

$16^{\frac{7}{4}} = \left(\sqrt[4]{16}\right)^7 = 2^7 = 128$

$(-27)^{\frac{4}{3}} = \left(\sqrt[3]{-27}\right)^4 = (-3)^4 = 81$

3. (7pts) Simplify. Express answers in terms of positive exponents.

$\sqrt[5]{64x^{10}y^{17}} =$ $\sqrt[5]{2^6 \cdot (x^2)^5 y^{15} y^2} = \sqrt[5]{2^5 \cdot 2 (x^2)^5 (y^3)^5 y^2} = 2x^2 y^3 \sqrt[5]{2y^2}$

$$\frac{\left(4x^{\frac{1}{3}}y^{\frac{3}{4}}\right)^{\frac{5}{2}}}{\left(6x^{-\frac{2}{3}}y^{\frac{1}{8}}\right)^2} = \frac{4^{\frac{5}{2}} x^{\frac{1}{3}\cdot\frac{5}{2}} y^{\frac{3}{4}\cdot\frac{5}{2}}}{6^2 x^{-\frac{2}{3}\cdot 2} y^{\frac{1}{8}\cdot 2}} = \frac{(\sqrt{4})^5 x^{\frac{5}{6}} y^{\frac{15}{8}}}{36\, x^{-\frac{4}{3}} y^{\frac{1}{4}}} = \frac{8}{36} x^{\frac{5}{6}-(-\frac{4}{3})} y^{\frac{15}{8}-\frac{1}{4}}$$

$$= \frac{8}{9} x^{\frac{5+8}{6}} y^{\frac{15-2}{8}} = \frac{8}{9} x^{\frac{13}{6}} y^{\frac{13}{8}}$$

4. (2pts) Rationalize the denominator.

$$\frac{3+5\sqrt{2}}{7-\sqrt{2}} = \frac{3+5\sqrt{2}}{7-\sqrt{2}} \cdot \frac{7+\sqrt{2}}{7+\sqrt{2}} = \frac{21+3\sqrt{2}+35\sqrt{2}+5\cdot\sqrt{2}^2}{7^2-(\sqrt{2})^2} = \frac{31+38\sqrt{2}}{47}$$

$49-2$

5. (5pts) Simplify.

$(3-2i)(7+5i) = 21+15i-14i-10i^2 = 31+i$

($i^2 = -1$)

$$\frac{-7+i}{5+2i} = \frac{-7+i}{5+2i} \cdot \frac{5-2i}{5-2i} = \frac{-35+14i+5i-2i^2}{5^2-(2i)^2} = \frac{-33+19i}{29}$$

$4i^2 = -4$

6. (2pts) Simplify and justify your answer.

$i^{218} = \underbrace{(i^4)}_{=1}{}^{54} \cdot i^2 = i^2 = -1$

$218 \div 4 = 54$, rem 2

$218 = 4 \cdot 54 + 2$