1. (4pts) Convert to scientific notation or a decimal number:

$0.00003467 =$ $8.983 \times 10^7 =$

2. (9pts) Use formulas to expand:

a) $(5x + 4)^2 =$

b) $(x^2 - 3y)(x^2 + 3y) =$

3. (12pts) Simplify and write the answer so all exponents are positive:

a) $(3x^2 y^{\frac{3}{2}})^5 x^{\frac{2}{3}} y^{-8} =$

b) $\dfrac{6x^{-2}(2y)^3}{(4x^{-5}y^3)^3} =$

4. (8pts) Simplify.

$$\frac{x-3}{x^2-25} - \frac{4}{x^2-8x+15} =$$

5. (12pts) Put the complex number into form $a+bi$.

a) $\dfrac{(4-i)(3+2i)}{i} =$

b) (explain) $i^{71} =$

6. (5pts) Solve the equation.

$$4x + 1 - 2(x-4) = 5(x-7) + 11$$

7. (8pts) Solve the equation.

$$4x^2 - 5x = x - 1$$

8. (8pts) Solve the equation by completing the square.

$x^2 + 12x - 9 = 0$

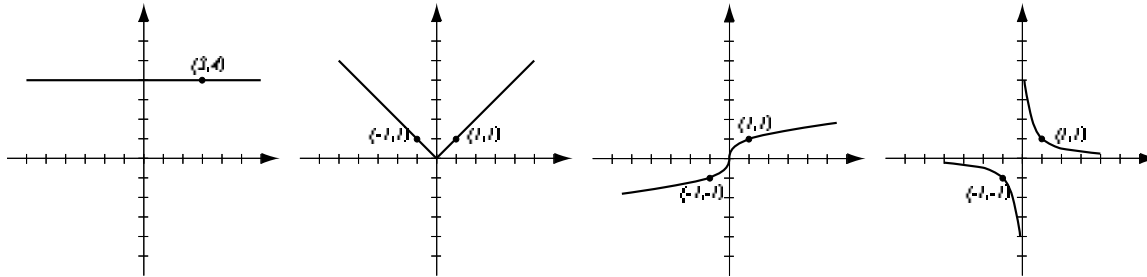**9.** (10pts) Solve the equation.

$x - 2 = 1 + \sqrt{23 - x}$

10. (12pts) A jogger travels a path in 40 minutes, while a walker takes an hour for the same path. How fast is each exerciser moving if the jogger runs 2mph faster than the walker? Write down the meaning of the variable you use.

11. (12pts) How many liters of water needs to be added to 8 liters of a 35% solution of hydrochloric acid in order to get a 15% solution? Write down the meaning of the variable you use.

Bonus (10pts) Write a quadratic equation in standard form $ax^2 + bx + c = 0$ whose solution set is $\{3 + \sqrt{5}, 3 - \sqrt{5}\}$. *Hint: start by doing the same problem if the desired solution set is* $\{1, -7\}$.

1. (8pts)   The following are graphs of basic functions. Write the equation of the graph under each one.

**2.** (14pts) Solve the inequalities and write the solution using interval notation:

$4 < 5 - 3x \leq 17$ $|x - 4| \geq 10$

3. (8pts) Write the equation of the circle centered at $(-1, 4)$ and passing through $(1, 3)$.

4. (8pts) Use the graph of the function f
at right to answer the following questions.
a) What is the domain of f?
b) What is the range of f?
c) Find $f(-6)$ and $f(3)$.
d) What are the solutions
    of the equation $f(x) = -1$?

(-3,4)

(-6,1)

1

-6 *-3 -2* *1 2 3* *5*

(3,-2)

-2

-4

(1,-5)
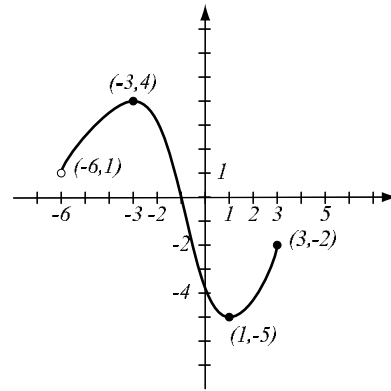
5. (12pts) Let $A = (1, 4)$ and $B = (-5, 2)$.
a) Find the midpoint M of A and B.
b) Find the slope of the line through A and B.
c) Write the equation of the line that passes through the midpoint M and is perpendicular
to the line through A and B.
d) Sketch a picture.

6. (24pts) Let $f(x) = -x^3 + 6x^2 - 5x + 9$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
c) Find the x- and y-intercepts.
d) Find where f has a local minimum and maximum.
e) Find the intervals of increase and decrease.
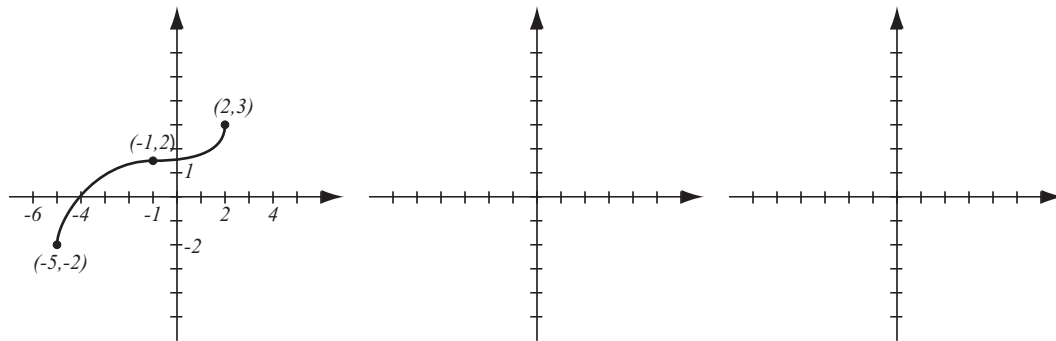f) Find all x for which $f(x) < 0$.

7. (6pts) Find the domain of the function $g(x) = \dfrac{7 - 4x}{5x + 6}$.

8. (10pts) Let $f(x) = 3x^2 - 4x + 7$, $g(x) = 2x - 5$. Determine the following and simplify where possible:

$f(2) =$ $g(\sqrt{a}) =$

$f(x + 3) - g(3x + 1) =$

9. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x - 2)$ and $1.5f(x)$ and label all the relevant points.

Bonus (10pts) Let $A = (1, 4)$ and $B = (-5, 2)$, as in problem 5. Show that all points P in the plane whose distance to A and B is equal form a line. Find the equation of this line and compare your answer to problem 5. (*Hint: let $P = (x, y)$, write $d(P, A) = d(P, B)$ using coordinates and simplify this equation.*)

1. (19pts) Let $f(x) = x^2 + 5x + 4$, $g(x) = \dfrac{2x+7}{x-1}$, $h(x) = \sqrt{5x-2}$.

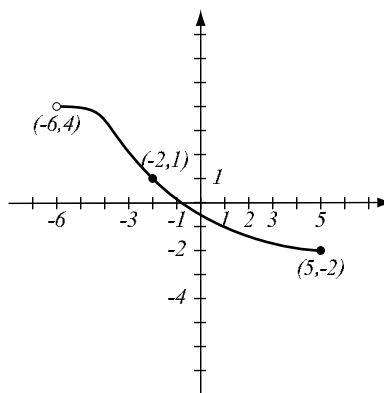Find the following (simplify where possible):

$\dfrac{f}{g}(0) =$ $\qquad\qquad\qquad\qquad\qquad\qquad\quad (f-h)(1) =$

$(f \circ g)(2)$ $\qquad\qquad\qquad\qquad\qquad\qquad\quad (g \circ f)(x) =$

The domain of $(g \cdot h)(x)$

2. (8pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.

3. (10pts) Let $f(x) = 5x - 4$.
a) Find the formula for f^{-1}.
b) Show that $(f^{-1} \circ f)(x) = x$.

4. (14pts) The quadratic function $f(x) = x^2 - 7x + 11$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

5. (24pts) Consider the polynomial $f(x) = x^3 - 8x^2 + 16x$.
a) Find the y- and x-intercepts algebraically. What are the multiplicities of the zeroes of f?
b) Use your calculator to draw the graph of the function (on paper!).
c) Find all the turning points (4 decimal points accuracy).
d) Describe the end behavior of f.
e) What is the range of f?

6. (10pts) Suppose you are fencing in a rectangular area for your goat. The width of the rectangle is 10 feet less than its length. Let P be the length (in feet) of fencing you bought.
a) Express the length l of the rectangle as a function of P.
b) Express the area A of the enclosure first as a function of length l, then as a function of P.

7. (15pts) A rectangle in the first quadrant is positioned as in the picture, so that two of its sides are along the axes, and one of its vertices is on the line $y = 5 - 2x$.
a) Draw two more such rectangles.
b) Express the area of the rectangle as a function of x and sketch a graph of the area function.
c) What dimensions of the rectangle give you the largest area, and what is this area?
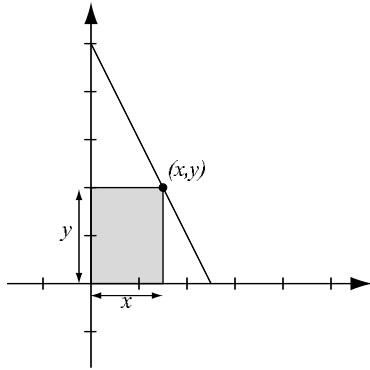
Bonus. (10pts) Let $f(x) = \dfrac{4x^2 - 8x - 140}{x^2 + 6x + 9}$.
a) Find the domain of f and the vertical asymptotes, if any.
b) Find all the x- and y-intercepts.
c) Use your calculator to draw the graph of the function (on paper!).
d) Find all the turning points.
e) Find the horizontal asymptote, if any.

The rules: you may use your book and notes on this take-home exam. Your work is to be entirely your own: you may not talk to anybody else about the exam problems. Turn the exam in by Thursday, December 3rd.

1. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \left(\frac{27x^5}{\sqrt[4]{y^5}} \right) =$$

$$\log_7 \left(\frac{x^2 - 2x - 8}{x^2 - 10x + 25} \right) =$$

2. (10pts) Solve the equation:

$$\log_4(5x) - \log_4(x + 2) = 2$$

3. (6pts) Find the domain of $f(x) = \log_6(3x - 7)$.

4. (10pts) Suppose you invest $3,000 at a 7% interest rate, compounded quarterly. How long will it take until your investment has value $5,000?

5. (12pts) Radium-226 has a half-life of 1600 years. How long will it take 5 grams of radium-226 to be reduced to 2 grams? Recall that the mass is given by $m(t) = m_0 e^{-rt}$. Find r first.

6. (8pts) Evaluate without using the calculator:

$\log_4 64 =$ $\qquad\qquad$ $\log_7 \frac{1}{49} =$ $\qquad\qquad$ $\log_{81} 9 =$ $\qquad\qquad$ $\ln \sqrt[4]{e^3} =$

7. (4pts) Use your calculator to find $\log_{18} 101$ with accuracy 4 decimal places. Show how you obtained your number.

8. (12pts) Solve the equations.

$3^{x^2 - x - 16} = 81$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $5^{x+2} = 8^x$

9. (6pts) Draw the general shape of the graph for these functions. Indicate the x- and y-intercepts. What are the horizontal or vertical asymptotes of the graphs?

$f(x) = b^x,\ b > 1$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $f(x) = \log_b x,\ b > 1.$

10. (10pts) Write as a single logarithm. Simplify if possible.

$\log(5x) - \log(25x^2) =$

$\frac{1}{2}\log_5(x-2) + \frac{1}{2}\log_5(x+2) - \frac{1}{2}\log_5(x^2-4) =$

11. (10pts) In 1998, the township of Chaffville had 1,328 inhabitants. Thanks to a new interstate passing near it, Chaffville grew to 3,117 inhabitants by 2005.
a) Write the function that describes the population of Chaffville t years after 1998, if it is of the form $N(t) = N_0 e^{rt}$. (Find the growth rate r.)
b) Use the function to estimate the size of the population in 2001.

Bonus (10pts) The number of wolves in a protected area where they were tracked is given by $N(t) = \frac{200}{1+24e^{-0.2t}}$, t years after beginning of tracking.
a) How many wolves were there at the beginning?
b) How long will it take until population is 100 wolves?
c) Sketch the graph of the function $N(t)$ and comment what happens to the population over a long period of time.

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.
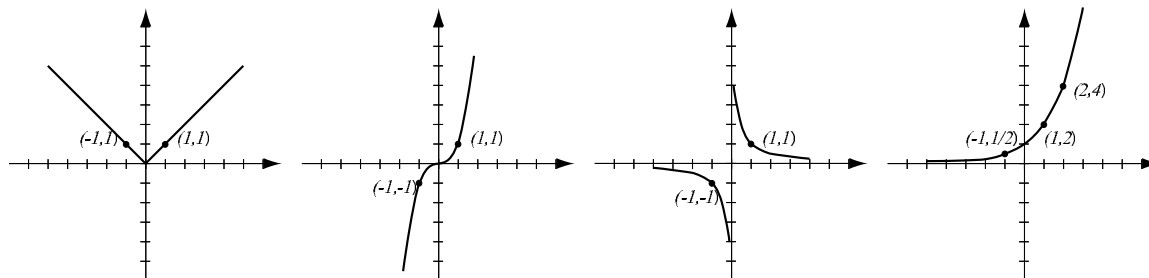
2. (10pts) Use the graph of the function f
at right to answer the following questions.
a) Find $f(1)$ and $f(-4)$.
b) What is the domain of f?
c) Is f one-to-one? Justify.
d) What are the solutions
   of the equation $f(x) = 2$?
e) Find the intervals where $f(x) < 0$.
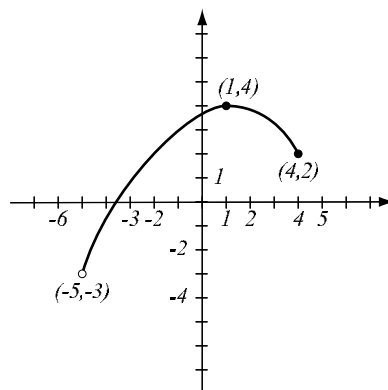
3. (7pts) Solve the inequality and write the solution using interval notation.

$|x + 7| < 4$

4. (9pts) The line $3x + 4y = 7$ is given.

a) Find the equation of the line that passes through $(-3, 2)$ and is parallel to the given line.

b) Sketch a picture of both lines.

5. (15pts) The quadratic function $f(x) = -x^2 - 8x + 8$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) What is the range of f?

6. (21pts) Consider the polynomial $f(x) = -x^3 + 7x$.

a) Find the y- and x-intercepts algebraically.

b) Use your calculator to draw the graph of the function (on paper!).

c) Find all the turning points (4 decimal points accuracy).

d) Describe the end behavior of f.

e) Find the intervals of increase.

f) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.

7. (6pts) Find the domain of the function $g(x) = \dfrac{\log_3(2x - 9)}{3x - 20}$.

8. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x) - 3$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.
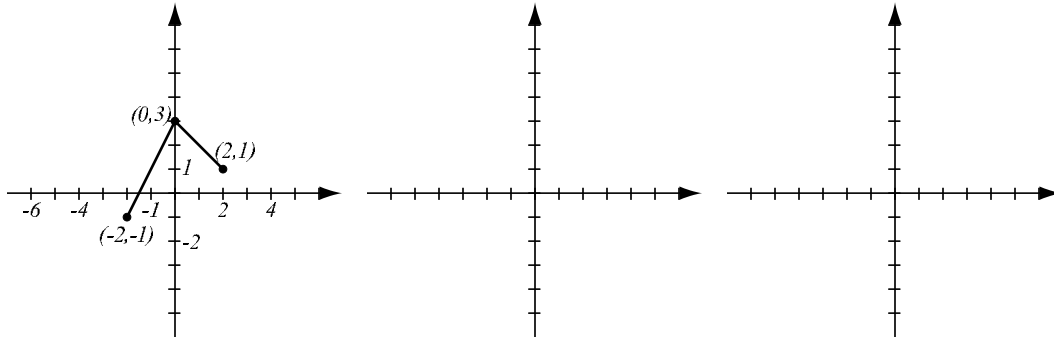
9. (10pts) Let $f(x) = 3x + 7$, $g(x) = \dfrac{5}{x-2}$. Find:

$(g \circ f)(x) =$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $g^{-1}(x) =$

10. (6pts) Simplify and write the answer so all exponents are positive:

$$\frac{(6x^{-3}y^6)^2}{\left(3x^{\frac{5}{2}}y^{-\frac{3}{2}}\right)^4} =$$

11. (8pts) Simplify.

$$\frac{3}{x^2 + 6x + 9} - \frac{2x+1}{x^2 - 4x - 21} =$$

12. (12pts) How many milliliters of a 10% solution of muriatic acid needs to be added to 200 milliliters of a 40% solution in order to get a 25% solution?

13. (4pts) Use your calculator to find $\log_7 3.6$ with accuracy 4 decimal places. Show how you obtained your number.

14. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_4 \frac{64}{y^7 \sqrt[3]{x^5}} =$$

15. (8pts) Solve the equation: $e^{3x+2} = 4^{x-4}$

16. (10pts) In 1998, the township of Chaffville had 1,328 inhabitants. Thanks to a new interstate passing near it, Chaffville grew to 3,117 inhabitants by 2005.

a) Write the function that describes the population of Chaffville t years after 1998, if it is of the form $N(t) = N_0 e^{rt}$. (Find the growth rate r.)

b) Use the function to estimate the size of the population in 2001.

Bonus (14pts) A rectangle in the first quadrant is positioned as in the picture, so that two of its sides are along the axes, and one of its vertices is on the line $y = 5 - 2x$.

a) Draw two more such rectangles.

b) Express the area of the rectangle as a function of x and sketch a graph of the area function.

c) What dimensions of the rectangle give you the largest area, and what is this area?