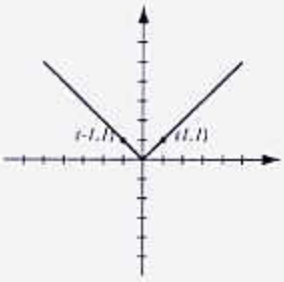
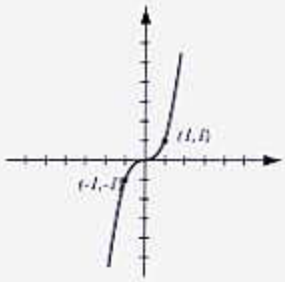
Fall '09/MAT 140/Final exam **Name:** Solution *Show all your work.*

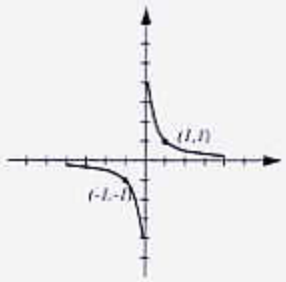
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



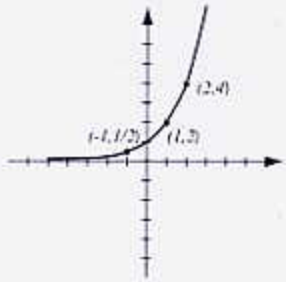
$f(x) = |x|$



$f(x) = x^3$



$f(x) = \frac{1}{x}$



$f(x) = 2^x$

2. (10pts) Use the graph of the function f at right to answer the following questions.

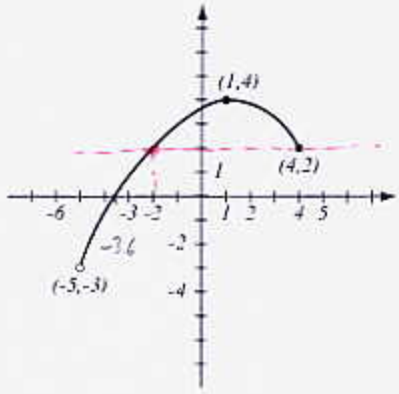
a) Find $f(1)$ and $f(-4)$.

b) What is the domain of f?

c) Is f one-to-one? Justify.

d) What are the solutions of the equation $f(x) = 2$?

e) Find the intervals where $f(x) < 0$.



a) $f(1) = 4$, $f(-4) \approx -0.5$

b) Domain $= (-5, 4]$

c) No. It fails the horizontal line test

d) $x = -2$, $x = 4$

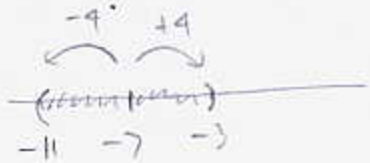
e) $f(x) < 0$ for x in $(-5, -3.6)$

3. (7pts) Solve the inequality and write the solution using interval notation.

$|x + 7| < 4$

$|x - (-7)| < 4$

distance from x to -7 < 4



$-11 < x < -3$

$(-11, -3)$

4. (9pts) The line $3x + 4y = 7$ is given.
a) Find the equation of the line that passes through $(-3, 2)$ and is parallel to the given line.
b) Sketch a picture of both lines.

a) $3x+4y=7$

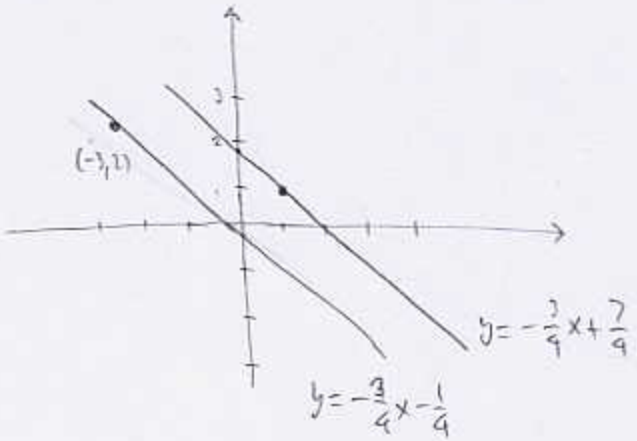
$4y=-3x+7 \quad \div 4$

$y=-\frac{3}{4}x+\frac{7}{4}$

slope $=-\frac{3}{4}$

$y-2=-\frac{3}{4}(x+3)$

$y=-\frac{3}{4}x-\frac{9}{4}+2=-\frac{3}{4}x-\frac{1}{4}$



5. (15pts) The quadratic function $f(x) = -x^2 - 8x + 8$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) What is the range of f?

a) y-int: $f(0)=8$

x-int $-x^2-8x+8=0$

$x^2+8x-8=0$ doesn't factor

$x=\frac{-8\pm\sqrt{64-4\cdot1\cdot(-8)}}{2}=\frac{-8\pm\sqrt{96}}{2}$ $16\cdot6$

$=\frac{-8\pm4\sqrt{6}}{2}=-4\pm2\sqrt{6}$

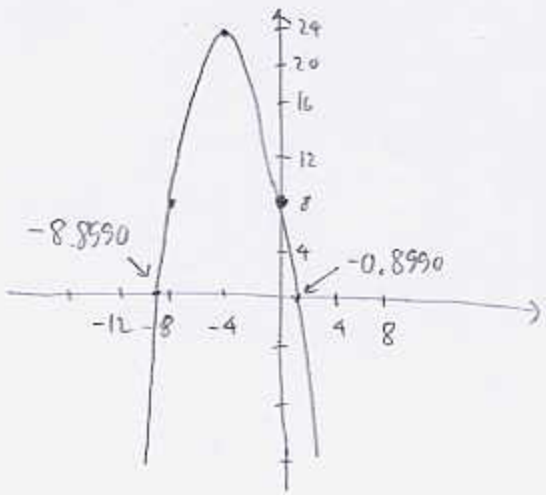
$\approx 0.8990, -8.8990$

b) $h=-\frac{-8}{2\cdot(-1)}=-4$

$k=-(-4)^2-8(-4)+8=-16+32+8$

$=24$

c)



d) Range $=(-\infty, 24]$

6. (21pts) Consider the polynomial $f(x) = -x^3 + 7x$.
a) Find the y- and x-intercepts algebraically.
b) Use your calculator to draw the graph of the function (on paper!).
c) Find all the turning points (4 decimal points accuracy).
d) Describe the end behavior of f.
e) Find the intervals of increase.
f) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.

a) y-int: $f(0) = 0$

x-int: $-x^3 + 7x = 0$

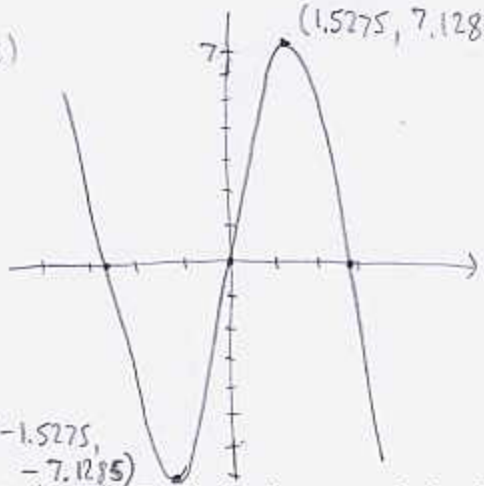
$x(-x^2 + 7) = 0$

$x = 0$ or $x^2 - 7 = 0$

$x^2 = 7$

$x = \pm\sqrt{7} \approx \pm 2.6458$

b)



c) f has a local max at $x = 1.5275$ with value $y = 7.1285$

f has a local min at $x = -1.5275$ with value $y = -7.1285$

d) It behaves like $-x^3$

e) f is increasing on $(-1.5275, 1.5275)$

f) $f(-x) = -(-x)^3 + 7(-x)$

$= -(-x^3) - 7x = x^3 - 7x = -(-x^3 + 7x)$

$= -f(x)$

f is odd, as can be seen from symmetry about the origin of the graph.

7. (6pts) Find the domain of the function $g(x) = \dfrac{\log_3(2x - 9)}{3x - 20}$.

Must have:

$2x - 9 > 0$ $\quad$ $3x - 20 \neq 0$

$2x > 9$ $\quad$ $3x - 20 = 0$

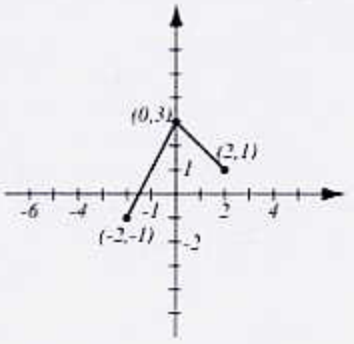
$x > \frac{9}{2}$ $\quad$ $3x = 20$

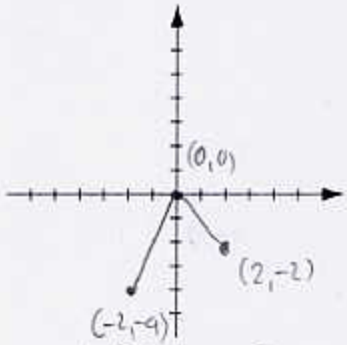
$x = \frac{20}{3}$

Domain $= \{x \mid x > \frac{9}{2} \text{ and } x \neq \frac{20}{3}\}$

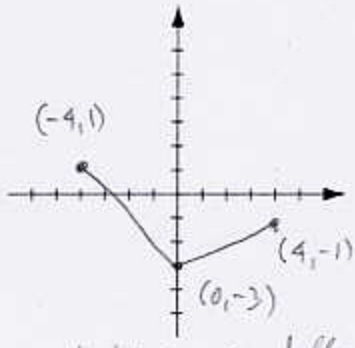
$(\frac{9}{2}, \frac{20}{3}) \cup (\frac{20}{3}, \infty)$

8. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x) - 3$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.





shift down 3



stretch horizontally, factor $= \frac{1}{\frac{1}{2}} = 2$
reflect over x-axis

9. (10pts) Let $f(x) = 3x + 7$, $g(x) = \dfrac{5}{x-2}$. Find:

$(g \circ f)(x) = g(f(x))$

$= g(3x+7)$

$= \dfrac{5}{3x+7-2} = \dfrac{5}{3x+5}$

$g^{-1}(x) = \dfrac{2x+5}{x}$

$y = \dfrac{5}{x-2}$ solve for x

$y(x-2) = 5$

$yx - 2y = 5$

$yx = 2y + 5$

$x = \dfrac{2y+5}{y} = g^{-1}(y)$

10. (6pts) Simplify and write the answer so all exponents are positive:

$$\frac{(6x^{-3}y^6)^2}{\left(3x^{\frac{5}{2}}y^{-\frac{3}{2}}\right)^4} = \frac{36x^{-6}y^{12}}{81x^{10}y^{-6}} = \frac{4 \cdot 9\, y^{12}y^6}{9 \cdot 9\, x^{10}x^6} = \frac{4y^{18}}{9x^{16}}$$

$\frac{5}{2} \cdot 4 = 10 \quad -\frac{3}{2} \cdot 4 = -6$

11. (8pts) Simplify.

$$\frac{3}{x^2+6x+9} - \frac{2x+1}{x^2-4x-21} = \frac{3}{(x+3)^2} - \frac{2x+1}{(x+3)(x-7)} = \frac{3(x-7) - (2x+1)(x+3)}{(x+3)^2(x-7)}$$

$$= \frac{3x - 21 - (2x^2+7x+3)}{(x+3)^2(x-7)} = \frac{-2x^2-4x-24}{(x+3)^2(x-7)} = -\frac{2(x^2+2x+12)}{(x+3)^2(x-7)}$$

prod = 12
sum = 2
cannot factor

$\pm 1, 12$	$\pm 2, 6$	$\pm 3, 4$
13	± 8	± 7

sum: -13

12. (12pts) How many milliliters of a 10% solution of muriatic acid needs to be added to 200 milliliters of a 40% solution in order to get a 25% solution?

Compare amount of pure muriatic acid

$$0.1x + 0.4 \cdot 200 = 0.25(x + 200)$$

$$0.1x + 80 = 0.25x + 50 \qquad 1 = \frac{0.1x}{50}$$

$$30 = 0.15x$$

$$x = \frac{30}{0.15} = 200 \text{ ml}$$

13. (4pts) Use your calculator to find $\log_7 3.6$ with accuracy 4 decimal places. Show how you obtained your number.

$$\log_7 3.6 = \frac{\log 3.6}{\log 7} = 0.6583$$

14. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_4 \frac{64}{y^7 \sqrt[3]{x^5}} = \log_4 64 - \log_4 (y^7 x^{\frac{5}{3}})$$

$$= 3 - (\log_4 y^7 + \log_4 x^{\frac{5}{3}}) = 3 - 7\log_4 y - \frac{5}{3}\log_4 x$$

$4^3 = 64$

15. (8pts) Solve the equation: $e^{3x+2} = 4^{x-4}$

$$e^{3x+2} = 4^{x-4} \quad | \ln$$

$$\ln e^{3x+2} = \ln 4^{x-4}$$

$$3x+2 = (x-4)\ln 4$$

$$3x+2 = x\ln 4 - 4\ln 4$$

$$3x - x\ln 4 = -2 - 4\ln 4$$

$$x(3 - \ln 4) = -2 - 4\ln 4$$

$$x = -\frac{2 + 4\ln 4}{3 - \ln 4} \approx -4.6757$$

16. (10pts) In 1998, the township of Chaffville had 1,328 inhabitants. Thanks to a new interstate passing near it, Chaffville grew to 3,117 inhabitants by 2005.
a) Write the function that describes the population of Chaffville t years after 1998, if it is of the form $N(t) = N_0 e^{rt}$. (Find the growth rate r.)
b) Use the function to estimate the size of the population in 2001.

a) $3117 = 1328\, e^{7r} \quad | \div 1328$

$\frac{3117}{1328} = e^{7r} \quad | \ln$

$\ln \frac{3117}{1328} = 7r$

$r = \frac{\ln \frac{3117}{1328}}{7} = 0.121885$

b) Need $N(3)$

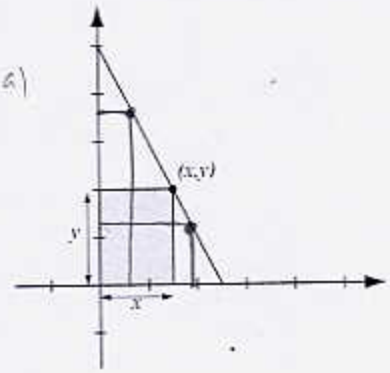
$N(3) = 1328\, e^{0.121885 \cdot 3}$

$= 1914.2576$

It will have about 1914 inhabitants in 2001

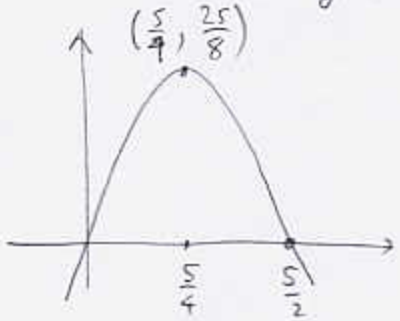
Bonus (14pts) A rectangle in the first quadrant is positioned as in the picture, so that two of its sides are along the axes, and one of its vertices is on the line $y = 5 - 2x$.
a) Draw two more such rectangles.
b) Express the area of the rectangle as a function of x and sketch a graph of the area function.
c) What dimensions of the rectangle give you the largest area, and what is this area?

a)



b) $A = x \cdot y = x(5-2x) = -2x^2 + 5x$ (graph: parabola opening down)

↑ since part is a line, $y = 5 - 2x$



$x(5-2x) = 0$

$x = 0$ or $5 - 2x = 0$

$x = \frac{5}{2}$

c) Area is maximal at $x = \frac{5}{4}$, it is $A = \frac{5}{4} \cdot (5 - 2 \cdot \frac{5}{4})$

$= \frac{5}{4} \cdot \frac{5}{2} = \frac{25}{8}$