Fall '09/MAT 140/Exam 4, take-home part **Name:** Solution *Show all your work.*

The rules: you may use your book and notes on this take-home exam. Your work is to be entirely your own: you may not talk to anybody else about the exam problems. Turn the exam in by Thursday, December 3rd.

1. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3\left(\frac{27x^5}{\sqrt[4]{y^5}}\right) = \log_3(27x^5) - \log_3(y^{\frac{5}{4}})$$

$$= \log_3 27 + \log_3 x^5 - \log_3 y^{\frac{5}{4}}$$

$$= 3 + 5\log_3 x - \frac{5}{4}\log_3 y$$

$$\log_7\left(\frac{x^2-2x-8}{x^2-10x+25}\right) = \log_7\left(\frac{(x-4)(x+2)}{(x-5)^2}\right) = \log_7((x-4)(x+2)) - \log_7(x-5)^2$$

$$= \log_7(x-4) + \log_7(x+2) - 2\log_7(x-5)$$

2. (10pts) Solve the equation:

$\log_4(5x) - \log_4(x+2) = 2$

$$\log_4 \frac{5x}{x+2} = 2$$

$$4^{\log_4 \frac{5x}{x+2}} = 4^2$$

$$\frac{5x}{x+2} = 16 \quad \Big| \cdot x+2$$

$$5x = 16(x+2)$$

$$5x = 16x + 32 \quad \Big| \begin{matrix}-5x \\ -32\end{matrix}$$

$$-32 = 11x$$

$$x = -\frac{32}{11}$$

cannot plug back into original:

$$\underbrace{\log_4\left(5\cdot\left(-\frac{32}{11}\right)\right)}_{<0} - \log_4\left(-\frac{32}{11}+2\right) = 2$$

so $\log_4$ not defined

No solution.

3. (6pts) Find the domain of $f(x) = \log_6(3x - 7)$.

Must have

$3x - 7 > 0$

$3x > 7$

$x > \frac{7}{3}$

Domain $= \left(\frac{7}{3}, \infty\right)$

$\frac{7}{3}$

4. (10pts) Suppose you invest \$3,000 at a 7% interest rate, compounded quarterly. How long will it take until your investment has value \$5,000?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$5000 = 3000\left(1 + \frac{0.07}{4}\right)^{4t} \quad \Big| \div 3000$

$\frac{5}{3} = (1.0175)^{4t} \quad \Big| \ln$

$\ln\frac{5}{3} = \ln(1.0175)^{4t}$

$\ln\frac{5}{3} = 4t\ln(1.0175) \quad \Big| \div 4\ln(1.0175)$

$t = \dfrac{\ln\frac{5}{3}}{4\ln(1.0175)} = 7.3612$ years

5. (12pts) Radium-226 has a half-life of 1600 years. How long will it take 5 grams of radium-226 to be reduced to 2 grams? Recall that the mass is given by $m(t) = m_0e^{-rt}$. Find r first.

$\frac{m_0}{2} = m_0 e^{-r \cdot 1600} \quad \Big| \div m_0$

$\frac{1}{2} = e^{-1600r} \quad \Big| \ln$

$\ln\frac{1}{2} = \ln e^{-1600r}$

$\ln\frac{1}{2} = -1600r$

$r = \dfrac{\ln\frac{1}{2}}{-1600} = 0.000433217$

$2 = 5e^{-0.00043\_t}$

$\frac{2}{5} = e^{-0.00043\_t} \quad \Big| \ln$

$\ln\frac{2}{5} = -0.00043\_t$

$t = \dfrac{\ln\frac{2}{5}}{-0.000433217} = 2115.0850$ years

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6. (8pts) Evaluate without using the calculator:

$\log_4 64 = 3$ $\quad\quad$ $\log_7 \frac{1}{49} = -2$ $\quad\quad$ $\log_{81} 9 = \frac{1}{2}$ $\quad\quad$ $\ln \sqrt[4]{e^3} = \frac{3}{4}$

$4^{?} = 64$ $\quad\quad$ $7^{?} = \frac{1}{49}$ $\quad\quad$ $81^{?} = 9$ $\quad\quad$ $e^{?} = e^{\frac{3}{4}}$

7. (4pts) Use your calculator to find $\log_{18} 101$ with accuracy 4 decimal places. Show how you obtained your number.

$$\log_{18} 101 = \frac{\ln 101}{\ln 18} = 1.5967$$

8. (12pts) Solve the equations.

$3^{x^2-x-16} = 81$

$3^{x^2-x-16} = 3^4$

$x^2 - x - 16 = 4$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = -4, 5$

$5^{x+2} = 8^x \quad | \ln$

$\ln 5^{x+2} = \ln 8^x$

$(x+2)\ln 5 = x \ln 8$

$x\ln 5 + 2\ln 5 = x \ln 8$

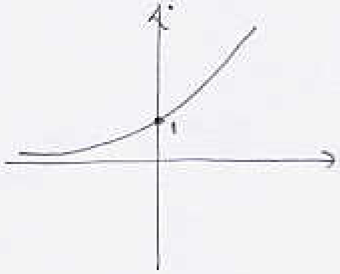
$2\ln 5 = x\ln 8 - x\ln 5$

$2\ln 5 = x(\ln 8 - \ln 5)$

$x = \frac{2\ln 5}{\ln 8 - \ln 5} = 6.8486$

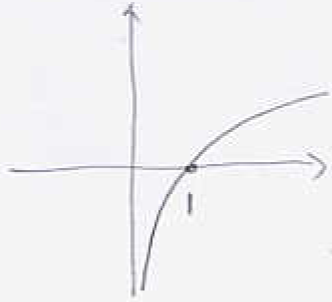
9. (6pts) Draw the general shape of the graph for these functions. Indicate the x- and y-intercepts. What are the horizontal or vertical asymptotes of the graphs?

$f(x) = b^x,\ b > 1$



horizontal asymptote: x-axis

$f(x) = \log_b x,\ b > 1.$



vertical asymptote: y-axis

10. (10pts) Write as a single logarithm. Simplify if possible.

$\log(5x) - \log(25x^2) =$ $\log \frac{\cancel{5x}}{\cancel{25}x^{\cancel{2}}} = \log \frac{1}{5x}$

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$\frac{1}{2}\log_5(x-2) + \frac{1}{2}\log_5(x+2) - \frac{1}{2}\log_5(x^2-4) =$ $\log_5 (x-2)^{\frac{1}{2}}(x+2)^{\frac{1}{2}} - \log_5 (x^2-4)^{\frac{1}{2}}$

$= \log_5 \frac{\sqrt{x-2}\sqrt{x+2}}{\sqrt{x^2-4}} = \log_5 \frac{\sqrt{x^2-4}}{\sqrt{x^2-4}} = \log_5 1 = 0$

11. (10pts) In 1998, the township of Chaffville had 1,328 inhabitants. Thanks to a new interstate passing near it, Chaffville grew to 3,117 inhabitants by 2005.
a) Write the function that describes the population of Chaffville t years after 1998, if it is of the form $N(t) = N_0e^{rt}$. (Find the growth rate r.)
b) Use the function to estimate the size of the population in 2001.

a) $3117 = 1328\, e^{r \cdot 7}$ ← (years from 1998 to 2007)

$\frac{3117}{1328} = e^{7r}$ | ln

$\ln \frac{3117}{1328} = 7r$

$r = \frac{\ln \frac{3117}{1328}}{7} = 0.121885$

b) $2001 \leftrightarrow t = 3$

$N(3) = 1328 \cdot e^{3 \cdot 0.121885}$

$= 1914.2576$

About 1914 inhabitants in 2001

Bonus (10pts) The number of wolves in a protected area where they were tracked is given by $N(t) = \frac{200}{1+24e^{-0.2t}}$, t years after beginning of tracking.
a) How many wolves were there at the beginning?
b) How long will it take until population is 100 wolves?
c) Sketch the graph of the function $N(t)$ and comment what happens to the population over a long period of time.

a) $N(0) = \frac{200}{1+24 \cdot 1} = 8$

b) $100 = \frac{200}{1+24e^{-0.2t}}$ | $\div 200$

$\frac{1}{2} = \frac{1}{1+24e^{-0.2t}}$ | take reciprocals

$1+24e^{-0.2t} = 2$

$24e^{-0.2t} = 1$

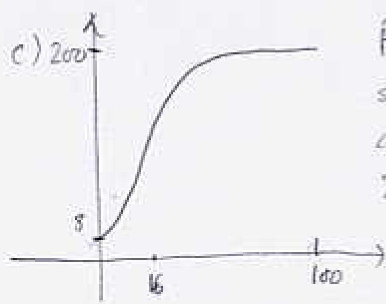
$e^{-0.2t} = \frac{1}{24}$ | ln

$-0.2t = \ln \frac{1}{24}$

$t = \frac{\ln \frac{1}{24}}{-0.2} = 15.8903$

About 16 years to reach 100 wolves

c)



Population stabilizes at about 200 wolves