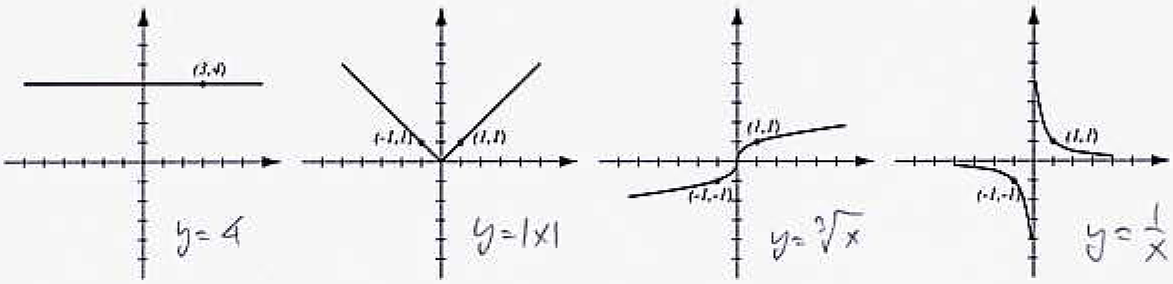
Fall '09/MAT 140/Exam 2 **Name:** Solution *Show all your work.*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=4$ $\quad$ $y=|x|$ $\quad$ $y=\sqrt[3]{x}$ $\quad$ $y=\frac{1}{x}$

2. (14pts) Solve the inequalities and write the solution using interval notation:

$4 < 5 - 3x \leq 17$ $\quad |-5$

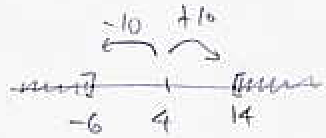
$-1 < -3x \leq 12 \quad |\div -3$

$\frac{1}{3} > x \geq -4$

$[-4, \frac{1}{3})$

$|x-4| \geq 10$

distance from x to $4 \geq 10$



$x \leq -6$ or $x \geq 14$

$(-\infty, 6] \cup [14, \infty)$

3. (8pts) Write the equation of the circle centered at $(-1,4)$ and passing through $(1,3)$).

C $\quad$ P

$r = d(C,P) = \sqrt{(1-(-1))^2+(3-4)^2} = \sqrt{4+1} = \sqrt{5}$

d

$(x-(-1))^2 + (y-4)^2 = (\sqrt{5})^2$

$(x+1)^2 + (y-4)^2 = 5$

4. (8pts) Use the graph of the function f at right to answer the following questions.

a) What is the domain of f? $(-6,3]$

b) What is the range of f? $[-5,4]$

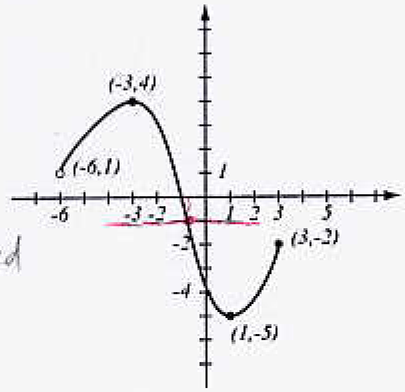
c) Find $f(-6)$ and $f(3)$.

d) What are the solutions of the equation $f(x) = -1$?

$f(-6)$ not defined

$f(3) = -2$

$x = -0.75$



5. (12pts) Let $A = (1,4)$ and $B = (-5,2)$.

a) Find the midpoint M of A and B.

b) Find the slope of the line through A and B.

c) Write the equation of the line that passes through the midpoint M and is perpendicular to the line through A and B.

d) Sketch a picture.

a) $M = \left(\frac{1-5}{2}, \frac{4+2}{2}\right) = (-2,3)$

b) slope $= \dfrac{2-4}{-5-1} = \dfrac{-2}{-6} = \dfrac{1}{3}$

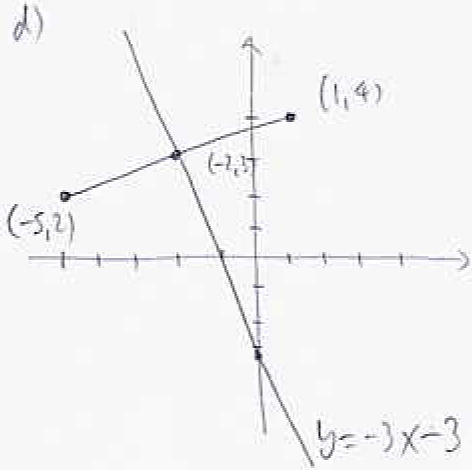
c) Slope of line is $-\dfrac{1}{\frac{1}{3}} = -3$

$y - 3 = -3(x-(-2))$

$y - 3 = -3x - 6$

$y = -3x - 3$

d)



6. (24pts) Let $f(x) = -x^3 + 6x^2 - 5x + 9$ (answer with 4 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.

b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.

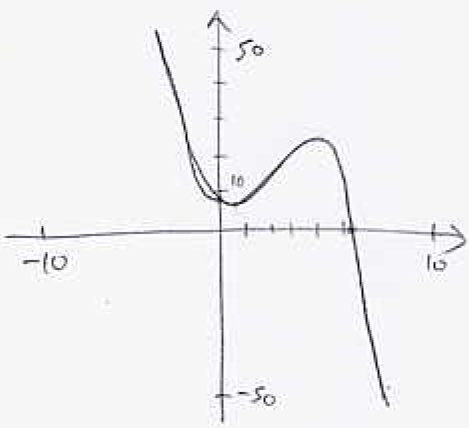
c) Find the x- and y-intercepts.

d) Find where f has a local minimum and maximum.

e) Find the intervals of increase and decrease.

f) Find all x for which $f(x) < 0$.

a)



c) y-int $f(0) = 9$

x-int: $x = 5.3817$

d) f has a local min at $x = 0.4725$

with value $y = 7.8715$

f has a local max at $x = 3.5275$

with value $y = 22.1285$

e) incr. on $(0.4725, 3.5275)$

decr. on $(-\infty, 0.4725) \cup (3.5275, \infty)$

b) $f(-x) = -(-x)^3 + 6(-x)^2 - 5(-x) + 9$

$= -(-x^3) + 6x^2 + 5x + 9$

$= x^3 + 6x^2 + 5x + 9$

$\neq f(x), \neq -f(x)$ so neither

Can see graph is not symmetric unto either y-axis or origin.

f) Graph is under x-axis for x in

$(5.3817, \infty)$

7. (6pts) Find the domain of the function $g(x) = \dfrac{7-4x}{5x+6}$.

Cannot have $5x + 6 = 0$

$5x = -6$

$x = \frac{-6}{5}$

Domain $= \{x \mid x \neq -\frac{6}{5}\}$

8. (10pts) Let $f(x) = 3x^2 - 4x + 7$, $g(x) = 2x - 5$. Determine the following and simplify where possible:

$f(2) = 3 \cdot 4 - 4 \cdot 2 + 7 = 11$ $\qquad g(\sqrt{a}) = 2\sqrt{a} - 5$

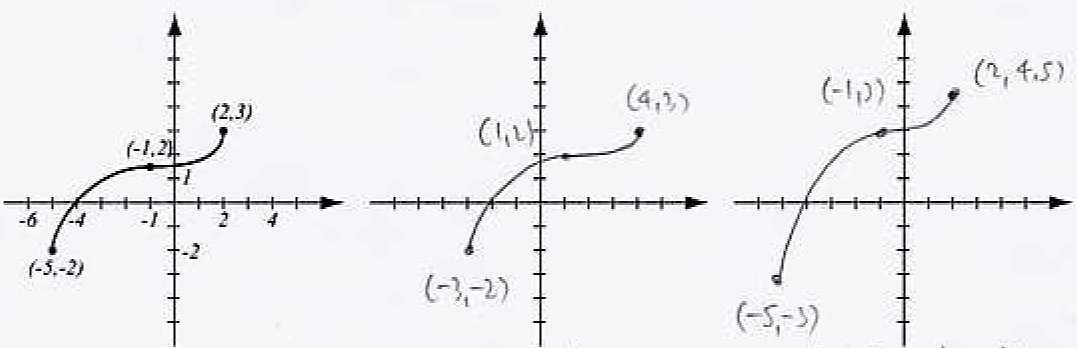
$f(x+3) - g(3x+1) = 3(x+3)^2 - 4(x+3) + 7 - (2(3x+1) - 5)$

$= 3(x^2 + 6x + 9) - 4x - 12 + 7 - (6x + 2 - 5)$

$= 3x^2 + 18x + 27 - 4x - 5 - 6x + 3$

$= 3x^2 + 8x + 25$

9. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x-2)$ and $1.5f(x)$ and label all the relevant points.



shift right 2 $\qquad$ stretch vertically by factor 1.5

Bonus (10pts) Let $A = (1, 4)$ and $B = (-5, 2)$, as in problem 5. Show that all points P in the plane whose distance to A and B is equal form a line. Find the equation of this line and compare your answer to problem 5. *(Hint: let $P = (x, y)$, write $d(P, A) = d(P, B)$ using coordinates and simplify this equation.)*

$d(P, A) = d(P, B)$

$\sqrt{(x-1)^2 + (y-4)^2} = \sqrt{(x-(-5))^2 + (y-2)^2}$ | square

$(x-1)^2 + (y-4)^2 = (x+5)^2 + (y-2)^2$

$\cancel{x^2} - 2x + 1 + \cancel{y^2} - 8y + 16 = \cancel{x^2} + 10x + 25 + \cancel{y^2} - 4y + 4$

$-2x - 8y + 17 = 10x - 4y + 29$

$-4y = 12x + 12 \quad | \div(-4)$

$y = -3x - 3$ $\leftarrow$ same answer, as it is the line perpendicular to AB, passing through the midpoint.