Fall '09/MAT 140/Exam 1 Name: Solution *Show all your work.*

1. (4pts) Convert to scientific notation or a decimal number:

$0.00003467 =$ 3.467×10^{-5} $8.983 \times 10^{7} =$ $89,830,000$

2. (9pts) Use formulas to expand:

a) $(5x+4)^2 =$ $(5x)^2 + 2 \cdot 5x \cdot 4 + 4^2 = 25x^2 + 40x + 16$

b) $(x^2-3y)(x^2+3y) =$ $(x^2)^2 - (3y)^2 = x^4 - 9y^2$

3. (12pts) Simplify and write the answer so all exponents are positive:

a) $(3x^2y^{\frac{3}{2}})^5 x^{\frac{2}{3}} y^{-8} =$ $3^5 x^{2 \cdot 5} y^{\frac{3}{2} \cdot 5} x^{\frac{2}{3}} y^{-8} = 243 x^{10} y^{\frac{15}{2}} x^{\frac{2}{3}} y^{-8}$

$= 243 x^{10+\frac{2}{3}} y^{\frac{15}{2}-8} = 243 x^{\frac{32}{3}} y^{-\frac{1}{2}} = \dfrac{243 x^{\frac{32}{3}}}{y^{\frac{1}{2}}}$

b) $\dfrac{6x^{-2}(2y)^3}{(4x^{-5}y^3)^3} =$ $\dfrac{6x^{-2}2^3y^3}{4^3 x^{-5 \cdot 3} y^{3 \cdot 3}} = \dfrac{\cancel{6}^{3} x^{-2} \cancel{8} y^3}{\cancel{64}^{\cancel{8}\,4} x^{-15} y^9} = \dfrac{3}{4} x^{-2+15} y^{3-9} = \dfrac{3}{4} x^{13} y^{-6} = \dfrac{3x^{13}}{4y^6}$

4. (8pts) Simplify.

$$\frac{x-3}{x^2-25}-\frac{4}{x^2-8x+15}=$$

$$\frac{x-3}{(x-5)(x+5)}-\frac{4}{(x-5)(x-3)}.$$

$$=\frac{(x-3)(x-3)-4(x+5)}{(x-5)(x+5)(x-3)}=\frac{x^2-6x+9-4x-20}{(x-5)(x+5)(x-3)}$$

$$=\frac{x^2-10x-11}{(x-5)(x+5)(x-3)}=\frac{(x-11)(x+1)}{(x-5)(x+5)(x-3)}$$

prod $=-11$ $\quad -11, 1$
sum $=-10$

5. (12pts) Put the complex number into form $a+bi$.

a) $\dfrac{(4-i)(3+2i)}{i}=\dfrac{12+8i-3i-2i^2}{i}=\dfrac{14+5i}{i}\cdot\dfrac{-i}{-i}=\dfrac{-14i-5\cdot i^2}{1}=5-14i$

b) (explain) $i^{71}=$ $i^{4\cdot17+3}=(i^4)^{17}i^3=i^3=-i$

$71\div4=17$, rem 3 $\quad 71=4\cdot17+3$

6. (5pts) Solve the equation.

$4x+1-2(x-4)=5(x-7)+11$

$4x+1-2x+8=5x-35+11$ $\quad x=\dfrac{33}{3}=11$

$2x+9=5x-24$

$33=3x$

7. (8pts) Solve the equation.

$4x^2-5x=x-1$

$4x^2-6x+1=0$

$36-16$ $\qquad 4\cdot5$

$$x=\frac{6\pm\sqrt{(-6)^2-4\cdot4\cdot1}}{2\cdot4}=\frac{6\pm\sqrt{20}}{8}$$

$$=\frac{6\pm2\sqrt{5}}{8}=\frac{2(3\pm\sqrt{5})}{8_4}=\frac{3\pm\sqrt{5}}{4}$$

8. (8pts) Solve the equation by completing the square. 9.5

$x^2 + 12x - 9 = 0$

$x^2 + 2x \cdot 6 = 9 \quad | + 6^2$

$x^2 + 2 \cdot x \cdot 6 + 6^2 = 9 + 36$

$(x+6)^2 = 45$

$x + 6 = \pm\sqrt{45}$

$x = -6 \pm 3\sqrt{5}$

9. (10pts) Solve the equation.

$x - 2 = 1 + \sqrt{23 - x} \quad | -1$

$x - 3 = \sqrt{23-x} \quad |^2$

$(x-3)^2 = 23 - x$

$x^2 - 6x + 9 = 23 - x$

$x^2 - 5x - 14 = 0$ prod $= -14$, sum $= -5$: $-7, 2$

$(x-7)(x+2) = 0$

$x = 7, -2$

$x = 7$ is solution

test: $7 - 2 \stackrel{?}{=} 1 + \sqrt{23-7}$

$5 = 1 + \sqrt{16}$

yes

$-2 - 2 \stackrel{?}{=} 1 + \sqrt{23-(-2)}$

$-4 \stackrel{?}{=} 1 + \sqrt{25}$

no

10. (12pts) A jogger travels a path in 40 minutes, while a walker takes an hour for the same path. How fast is each exerciser moving if the jogger runs 2mph faster than the walker? Write down the meaning of the variable you use.

$d = rt$ r = Walker's speed

jogger's trip $(40 \text{ min} = \frac{2}{3} \text{ hr})$

$d = (r+2) \cdot \frac{2}{3}$

walker's trip

$d = r \cdot 1$

Trips are equal, so: $(r+2)\frac{2}{3} = r \quad 1 \cdot 3$

$(r+2) \cdot 2 = 3r$

$2r + 4 = 3r$

$4 = r$

walker: 4 mph

jogger: 6 mph

11. (12pts) How many liters of water needs to be added to 8 liters of a 35% solution of hydrochloric acid in order to get a 15% solution? Write down the meaning of the variable you use.

x = liters of water added

water	+	35% sol.	=	15% sol.
x		8		$x+8$

no HCl in water $\longrightarrow$ $0 + 0.35 \cdot 8 = 0.15(x+8)$

$2.8 = 0.15x + 1.2 \quad | -1.2$

$1.6 = 0.15x$

$x = \frac{1.6}{0.15} = 10.6667$ liters

Bonus (10pts) Write a quadratic equation in standard form $ax^2 + bx + c = 0$ whose solution set is $\{3+\sqrt{5}, 3-\sqrt{5}\}$. *Hint: start by doing the same problem if the desired solution set is* $\{1, -7\}$.

If we want solution set to be $\{1,-7\}$ we form.

$(x-1)(x-(-7)) = 0$

$(x-1)(x+7) = 0$

$x^2 + 6x - 7 = 0$

Same idea for $\{3+\sqrt{5}, 3-\sqrt{5}\}$

$(x-(3+\sqrt{5}))(x-(3-\sqrt{5})) = 0$

$x^2 - (3-\sqrt{5})x - (3+\sqrt{5})x + (3+\sqrt{5})(3-\sqrt{5}) = 0$

$x^2 - 3x + \sqrt{5}x - 3x - \sqrt{5}x + 3^2 - \sqrt{5}^2 = 0$

$\boxed{x^2 - 6x + 4 = 0}$