Spring '08/MAT 250/Exam 1 **Name:** *Show all your work.*

1. (20pts) Find the following limits algebraically.

a) $\displaystyle\lim_{x \to -4} \frac{x^2 - x - 20}{x + 4} =$

b) $\displaystyle\lim_{x \to \infty} \frac{x^3 + 5x - 1}{(x^2 + 1)(x^2 - 3)} =$

c) $\displaystyle\lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5} =$

2. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

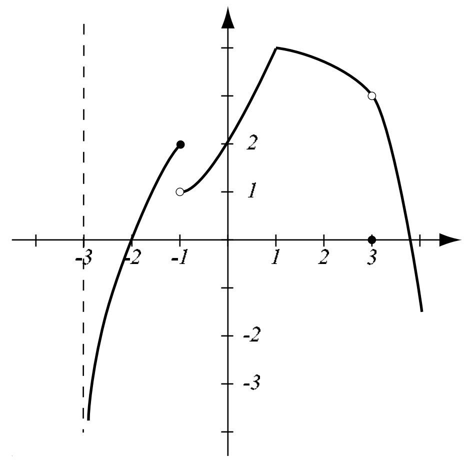
$\lim_{x \to -3^+} f(x) =$

$\lim_{x \to -1^-} f(x) =$

$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to 3} f(x) =$

List points where f is not continuous and justify why it is not continuous at those points.



At which point is f continuous but not differentiable? Why?

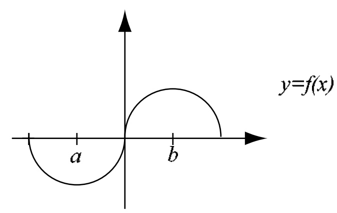
3. (16pts) The equation $x^3 = 7$ is given.
a) Use the Intermediate Value Theorem to show that this equation has at least one real solution. (Incindentally, this proves the existence of $\sqrt[3]{7}$).
b) Use your calculator to find an interval of width 0.01 that contains your solution. Use IVT again to justify why the interval you found contains the solution.

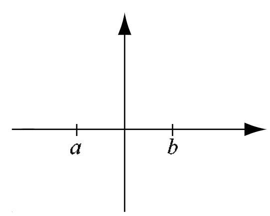
4. (10pts) Find $\lim_{x\to 0} \sin x \cos\left(\frac{1}{x} + \frac{1}{x^2}\right)$. Use the theorem that rhymes with what polite people say when requesting something.

5. (16pts) Let $f(x) = x^2 - 2x + 5$.
a) Find $f'(a)$.
b) Use a) to find the equation of the tangent line to the graph of f at the point $(2, 5)$.

6. (12pts) Let $f(x)$ denote the output in Watts of a square solar array whose side equals x meters.
a) What does $f'(x)$ represent and what are its units?
b) Suppose $f(3) = 120$ and $f'(3) = 11$. Use this information to estimate $f(3.2)$.

7. (10pts) The graph of $f(x)$ is given. Draw the graph of $f'(x)$ under the graph of $f(x)$.





Bonus. (10pts) Sketch the graph of a function that is continuous on $[-3, \infty)$ and satisfies the conditions given below.

x	-2	1	3
$f(x)$	2	-1	1

x	-2	1	3
$f'(x)$	-1	d.n.e.	0

$$\lim_{x \to \infty} f(x) = 0$$

Spring '08/MAT 250/Exam 2 **Name:** *Show all your work.*

Differentiate and simplify where appropriate:

1. (7pts) $\dfrac{d}{dx}\left(Ax^4 - \dfrac{1}{x^{10}} + \sqrt[4]{x^{20}} + \ln B\right) =$

2. (7pts) $\dfrac{d}{dx}\, e^{3x}\cos(5x) =$

3. (8pts) $\dfrac{d}{dx}\,\dfrac{3x-1}{x^2+x-3} =$

4. (7pts) $\dfrac{d}{dx}\,\arctan(\sqrt{x}) =$

5. (8pts) $\dfrac{d}{dx}\,\ln(\tan(x^4+x^2+1)) =$

6. (10pts) Use logarithmic differentiation to find $\dfrac{d}{dx}(\sin x)^{\cos x}$.

7. (14pts) Use implicit differentiation to find the equation of the tangent line to the curve $x^2 + 2xy - y^2 + x = 2$ at $(1, 2)$.

8. (10pts) Determine the following higher derivatives:

$D^{199}e^{3x} =$

$D^{58}\cos(5x) =$

9. (14pts) The position of a ball thrown upward with initial velocity 40 meters per second is given by $s(t) = -5t^2 + 40t$.

a) When does the ball reach its highest point and what is its altitude then?

b) When is the ball at height 60 meters? What is its velocity then?

10. (15pts) Luke Skywalker finds himself in a 4 meters wide rectangular garbage compactor containing 30m^3 of water. The side walls are closing in, causing its length to decrease at rate 1 meter per minute and the water level to rise (width stays constant). How fast is the depth of the water increasing when depth is 1.5 meters?

Bonus. (10pts) Let $f(x) = (x^2 + x)e^x$. Use the first several derivatives to find a general formula for the n-th derivative of $f(x)$.

Spring '08/MAT 250/Exam 3 **Name:** *Show all your work.*

1. (12pts) Draw the graph of a continuous function whose domain is $\mathbf{R}$ which satisfies the conditions below. (A sign chart may be helpful.)

$f'(x) > 0$ for $-2 < x < 1$, $f'(x) = 2$ for $x > 3$
$f'(x) < 0$ for $x < -2$, and $1 < x < 3$
$f'(3)$ does not exist
$f''(x) > 0$ for $x < \frac{1}{2}$
$f''(x) < 0$ for $\frac{1}{2} < x < 3$

2. (10pts) Use L'Hospital's rule to find the limit:

$$\lim_{x \to 0} \frac{\sin(2x) - 2x}{x^3} =$$

3. (10pts) Use linearization (or differentials) to estimate $\sqrt{4.3}$. By how much does your estimate differ from the actual value?

4. (14pts) Consider the function $f(x) = x^3 - 4x^2 + 2x + 7$ on the interval $[0, 1]$.
a) Verify the hypotheses of the Mean Value Theorem.
b) Verify the conclusion of the Mean Value Theorem.

5. (12pts) Find the absolute minimum and maximum values for the function $f(x) = x \ln x$ on the interval $[\frac{1}{5}, 5]$.

6. (24pts) Let $f(x) = \dfrac{1}{1+e^x}$.

a) Find the horizontal asymptotes.

b) Find the intervals of increase/decrease and where f has a local maximum and minimum.

c) Find the intervals where f is concave up or down and where it has an inflection point.

d) Use your calculator and the results of a), b) and c) to accurately sketch the graph of f.

7. (18pts) A rectangle is inscribed in the unit circle so that its sides are parallel to the axes.
a) Draw three pictures that illustrate some of the ways this can be done.
b) Among all such rectangles, find the dimensions of the one with the largest area.
Hints: let (x, y) *be the vertex of the rectangle that is in the first quadrant. Express the area of the rectangle using* x *and* y*. What is the connection between* x *and* y*?*

Bonus. (10pts) Find the limit.

$$\lim_{x\to\infty}\left(1+\frac{4}{x}\right)^{x} =$$

1. (20pts) The function $f(x) = e^x - 3$, $0 \le x \le 2$ is given.
a) Find the Riemann sum for the function with $n = 4$, taking sample points to be midpoints.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible.
c) What does the Riemann sum represent?
d) Using the Fundamental Theorem of Calculus, evaluate $\displaystyle\int_0^2 (e^x - 3)\,dx$. What is the error of the Riemann sum from b)?

2. (8pts) Write in sigma notation.

$$\frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \frac{7}{32} + \frac{8}{64} =$$

3. (4pts) Simplify using part 1 of the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_3^x \frac{\sin t}{t^2+1}\,dt =$$

4. (16pts) Find $\int_{-1}^{7}(5-x)\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Fundamental Theorem of Calculus.

5. (12pts) An old cell phone was thrown upwards. Find its position function $s(t)$ if at time $t=2$ its height was 12 meters, and its velocity was 1 meter per second. *Hint: a(t)=-10.*

Evaluate the following definite and indefinite integrals.

6. (8pts) (give exact value)

$$\int_{-\pi/3}^{\pi/4} \sin x \, dx =$$

7. (10pts) $\displaystyle\int x^2(\sqrt{x} + 3x^5)\, dx =$

8. (10pts) At time t, water is filling a tank at the rate $\frac{2}{t}$ liters per minute. Use the total change theorem to determine how much water is in the tank at time $t = 10$, if at time $t = 1$ it contained 25 liters.

9. (12pts) Use the substitution rule to evaluate the definite integral.

$$\int_1^3 \frac{6x+21}{(x^2+7x-5)^3}\,dx =$$

Bonus. (10pts) Let $g(x) = \displaystyle\int_0^x e^{\sin t}\,dt,\ 0 \le x \le 2\pi.$

a) Using g' and g'', find the intervals of increase/decrease and intervals of concavity of g.

b) Draw a nice graph of g.

1. (14pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim\limits_{x\to 0^-} f(x) =$

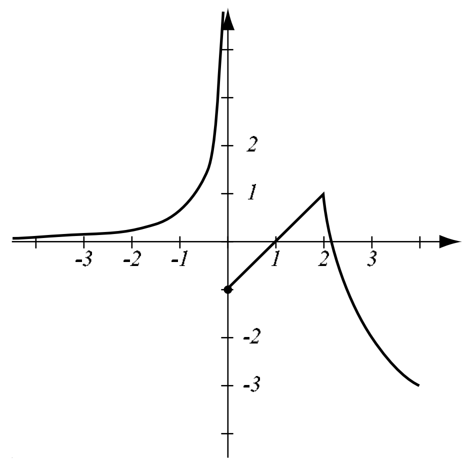
$\lim\limits_{x\to 0^+} f(x) =$

$\lim\limits_{x\to 0} f(x) =$

$\lim\limits_{x\to -\infty} f(x) =$

List points where f is not continuous and explain why.

List points where f is not differentiable and explain why.



2. (12pts) Find $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$ in two ways: a) algebraically b) Using L'Hospital's rule

3. (10pts) Find the absolute minimum and maximum values for the function $f(x) = x - 2\cos x$ on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

4. (10pts) Use implicit differentiation to find y'.

$$y \cos x = \frac{x}{y}$$

5. (14pts) Let $V(t) = t^3 - 5t^2 + 4t + 7$ be the volume, measured in liters, of water in a tank at time t minutes.

a) What is the volume at $t = 2$?

b) Find $V'(2)$. What does it represent? What are the units?

c) Use the numbers from a) and b) to approximate the volume at time $t = 2.2$.

d) What is the exact volume at time $t = 2.2$?

6. (21pts) Let $f(x) = x^2 e^x$.
a) Find the horizontal asymptotes.
b) Find the intervals of increase/decrease and where f has a local maximum and minimum.
c) Find the intervals where f is concave up or down and where it has an inflection point.
d) Use your calculator and the results of a), b) and c) to accurately sketch the graph of f.

7. (13pts) At time $t = 0$ a car starts moving westward from an intersection at speed 45mph. At time $t =$ 1hr another car starts moving southward from the same intersection, with speed 60mph. At what rate are the cars moving apart at time $t =$ 2hrs?

8. (12pts) Consider the integral $\int_4^6 x^2 - 7x + 10\, dx$.

a) Use a picture to determine whether this definite integral is positive or negative.

b) Evaluate the integral and verify your conclusion from a).

9. (6pts) Find $f(x)$ if $f'(x) = \frac{1}{x^5}$ and $f(1) = 3$.

10. (6pts) Find the indefinite integral:

$$\int \sqrt{x} - \sec^2 x \, dx =$$

11. (10pts) Use substitution (don't forget to change bounds) to evaluate:

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \cos^4 x \sin x \, dx =$$

12. (12pts) The equation $x + \sin x = 1$ is given.
a) Use the Intermediate Value Theorem to show that this equation has at least one real solution.
b) Use your calculator to find an interval of width 0.01 that contains your solution. Use IVT again to justify why the interval you found contains the solution.

Bonus. (14pts) Find the point on the graph of $y = e^x$ that is closest to the origin. Show that the point you find is, indeed, the closest. (Note: an equation will arise that you will not be able to solve algebraically, so use your calculator to get an approximate solution.)