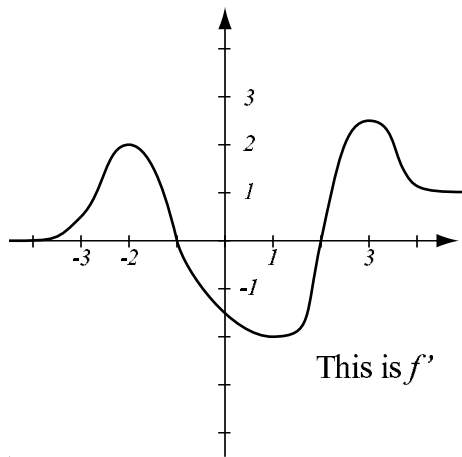**1.** (7pts) The graph of the function f' is given. Answer the following questions about f, which is defined for all real numbers (note: questions are not about f'). You may use a sign chart if it is helpful.

a) On which intervals is f increasing/decreasing?

b) On which intervals if f concave up/concave down?

c) At which points does f have local maxima/minima?

This is f'

2. (10pts) Use L'Hospital's rule to find the limits:

a) $\lim\limits_{x \to 0} \dfrac{e^x - 1 - x}{x^2} =$

b) $\lim\limits_{x \to 0^+} x^5 \ln x$

3. (7pts) Consider the function $f(x) = x^2 - 7x - 3$ on the interval $[2, 6]$.

a) Verify the hypotheses of the Mean Value Theorem.

b) Verify the conclusion of the Mean Value Theorem.

4. (7pts) Find the absolute minimum and maximum values for the function $f(x) = 4x - \tan x$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

5. (11pts) Let $f(x) = x^3 e^x$.

a) Find the intervals of increase/decrease and where f has a local maximum and minimum.

b) Find the intervals where f is concave up or down.

c) Use your calculator and the results of a) and b) to accurately sketch the graph of f.

6. (8pts) Sheila wishes to enclose a rectangular play pen so its area is 12m^2. Two sides of the pen are walls (see picture) and a fence is used for the remaining two sides. Find the dimensions of the pen that minimize the length of the fence. Show that the number you find, does, indeed, give you a minimum length.

wall

wall
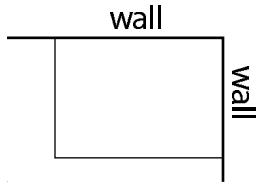
Bonus. (5pts) Use information you gathered in problem 1 to draw the graph of the function f if it is known that f satisfies the additional conditions:

$f(0) = 4$

$\lim\limits_{x \to -\infty} f(x) = 2$

What is $\lim\limits_{x \to \infty} f(x)$?