

Differentiate and simplify where appropriate:

1. (4pts)  $\frac{d}{dx} \left( 7x^7 - \frac{1}{\sqrt[4]{x^3}} - \frac{7}{x^4} + e \right) =$

2. (4pts)  $\frac{d}{dx} x^{10} e^{3x} =$

3. (4pts)  $\frac{d}{dx} \frac{x^2 + 4}{3x - 7} =$

4. (5pts)  $\frac{d}{dx} \ln \left( \frac{2x + 1}{3x - 7} \right)^4 =$

5. (5pts) Use logarithmic differentiation to find  $\frac{d}{dx} (x^2 + 3x - 1)^{\sin x}$ .

6. (4pts) Find the equation of the tangent line to the curve  $y = x^3 - 4x^2 + 7$  at the point  $(1, 4)$ .

7. (4pts) Find the first three derivatives of  $f(x)$  and use them to find the formula for  $f^{(n)}(x)$  if  $f(x) = \ln x$ .

8. (5pts) Use implicit differentiation to find  $y'$ .

$$\tan(xy) = 3x^2 + 5y^4$$

9. (8pts) A tank filled with 600 liters of water drains in 4 hours from an opening in the bottom. The volume of water in the tank after  $t$  hours is given by  $V(t) = 600(1 - \frac{t}{4})^2$ .

a) How much water is in the tank when  $t = 2$ ?

b) At what rate is the water draining when  $t = 2$ ? What are the units?

c) Interpret the meaning of the number in b) by approximating how much water there is in the tank at time  $t = 2.1$ .

d) What is the exact amount of water in the tank at time  $t = 2.1$ ?

**10.** (7pts) A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks from the spotlight to the wall at a speed of 1.6 meters per second, how fast is the length of his shadow on the wall decreasing when he is 8 meters away from the spotlight?

**Bonus.** (5pts) Let  $h(x) = f(x)g(x)$ . Find the formula for  $h''(x)$  in terms of  $f, f', f'', g, g', g''$ . What familiar formula from algebra does it resemble?