Spring '08/MAT 250/Exam 4 **Name:** Solution *Show all your work.*

1. (20pts) The function $f(x) = e^x - 3$, $0 \le x \le 2$ is given.
a) Find the Riemann sum for the function with $n = 4$, taking sample points to be midpoints.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible.
c) What does the Riemann sum represent?
d) Using the Fundamental Theorem of Calculus, evaluate $\int_0^2 (e^x - 3)\,dx$. What is the error of the Riemann sum from b)?

a) 0.25 0.75 1.25 1.75

0 2

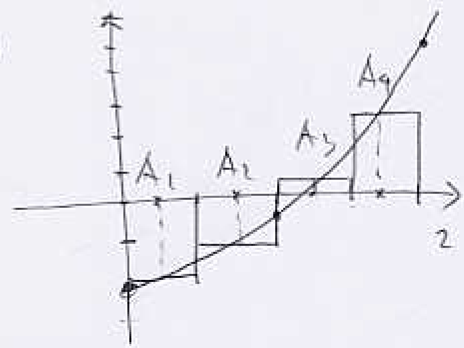
$\Delta x = \frac{1}{2}$

$M_4 = \frac{1}{2}\left(e^{0.25} - 3 + e^{0.75} - 3 + e^{1.25} - 3 + e^{1.75} - 1\right)$

$= \frac{1}{2}(12.645\ldots - 12)$

$= 0.322986$

b)



c) The Rieman sum is

$-A_1 - A_2 + A_3 + A_4$

d) $\int_0^2 e^x - 3\,dx = (e^x - 3x)\Big|_0^2 = e^2 - e^0 - 3(2-0)$
$= e^2 - 1 - 6 = e^2 - 7$

$e^2 - 7 = 0.389056$

$(e^2 - 7) - M_4 = 0.066070$

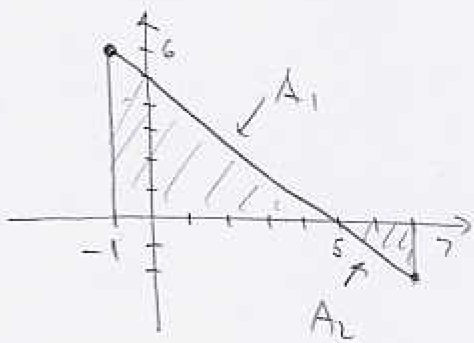
2. (8pts) Write in sigma notation.

$\frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \frac{7}{32} + \frac{8}{64} = \sum_{i=3}^{8} \frac{i}{2^{i-2}}$

3. (4pts) Simplify using part 1 of the Fundamental Theorem of Calculus:

$\frac{d}{dx}\int_3^x \frac{\sin t}{t^2+1}\,dt = \frac{\sin x}{x^2+1}$

4. (16pts) Find $\int_{-1}^{7}(5-x)\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Fundamental Theorem of Calculus.



a) $\int_{-1}^{7}(5-x)\,dx = A_1 - A_2 = \frac{1}{2}6.6 - \frac{1}{2}.2.2$

$= 18 - 2 = 16$

b) $\int_{-1}^{7}(5-x)dx = \left(5x - \frac{x^2}{2}\right)\Big|_{-1}^{7}$

$= 5(7-(-1)) - \frac{1}{2}(7^2 - (-1)^2)$

$= 5\cdot 8 - \frac{1}{2}(49-1)$

$= 40 - 24 = 16$

5. (~~10~~ 12pts) An old cell phone was thrown upwards. Find its position function $s(t)$ if at time $t = 2$ its height was 12 meters, and its velocity was 1 meter per second. *Hint: a(t)=-10.*

$a(t) = -10$

$v(t) = -10t + C$

$1 = v(2) = -10\cdot 2 + C$

$1 = -20 + C$

$C = 21$

$v(t) = -10t + 21 \quad \big| \int$

$s(t) = -10\frac{t^2}{2} + 21t + D$

$= -5t^2 + 21t + D$

$12 = s(2) = -5\cdot 2^2 + 21\cdot 2 + D$

$12 = -20 + 42 + D$

$-10 = D$

$s(t) = -5t^2 + 21t - 10$

Evaluate the following definite and indefinite integrals.

6. (8pts) (give exact value)

$$\int_{-\pi/3}^{\pi/4} \sin x\, dx = -\cos x \Big|_{-\pi/3}^{\pi/4} = -\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = \frac{1-\sqrt{2}}{2}$$

7. (~~12~~ 10pts) $\int x^2(\sqrt{x} + 3x^5)\, dx = \int x^{\frac{5}{2}} + 3x^7\, dx = \frac{2}{7} x^{\frac{7}{2}} + 3\frac{x^8}{8} + c$

$x^{\frac{1}{2}}$

8. (10pts) At time t, water is filling a tank at the rate $\frac{2}{t}$ liters per minute. Use the total change theorem to determine how much water is in the tank at time $t = 10$, if at time $t = 1$ it contained 25 liters.

Change in amount of water in tank

$$= V(10) - V(1) = \int_1^{10} V'(t)\, dt = \int_1^{10} \frac{2}{t}\, dt = 2\ln t \Big|_1^{10} = 2(\ln 10 - \ln 1) = 2\ln 10$$

$$V(10) = V(1) + \text{change} = 25 + 2\ln 10 = 29.605170$$

9. (12pts) Use the substitution rule to evaluate the definite integral.

$$\int_1^3 \frac{\overset{3(2x+7)}{6x+21}}{(x^2+7x-5)^3}\,dx = \begin{bmatrix} u = x^2+7x-5 & x=3,\ u=25 \\ du = 2x+7 & x=1,\ u=3 \end{bmatrix}$$

$$= \int_3^{25} \underbrace{\frac{3\,du}{u^3}}_{3u^{-3}} = 3\frac{u^{-2}}{-2}\Big|_3^{25} = -\frac{3}{2}\left(\frac{1}{25^2}-\frac{1}{3^2}\right)$$

$$= \frac{3}{2}\left(\frac{1}{9}-\frac{1}{625}\right) = 0.164267$$

$$= \frac{3}{2}\,\frac{625-9}{9\cdot 625} = \frac{3\cdot 616}{2\cdot 9\cdot 625} = \frac{308}{1875}$$

Bonus. (10pts) Let $g(x) = \int_0^x e^{\sin t}\,dt$, $0 \le x \le 2\pi$.

a) Using g' and g'', find the intervals of increase/decrease and intervals of concavity of g.

b) Draw a nice graph of g.

a) $g'(x) = e^{\sin x}$

$e^{\sin x} > 0$ so g is always increasing

$g''(x) = e^{\sin x}\cdot\cos x$

$\underbrace{e^{\sin x}}_{>0}\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

(on $[0, 2\pi]$)

	0	$\frac{\pi}{2}$		$\frac{3\pi}{2}$		2π
g''	+	0	−	0	+	
g	CU	IP	CD	IP	CU