Spring '08/MAT 250/Exam 2 **Name:** Solution *Show all your work.*

Differentiate and simplify where appropriate:

1. (7pts) $\frac{d}{dx}\left(Ax^4 - \frac{1}{x^{10}} + \sqrt[4]{x^{20}} + \ln B\right) = 4Ax^3 + 10x^{-11} + 5x^4$

x^{-10} $x^{\frac{20}{4}} = x^5$ constant

2. (7pts) $\frac{d}{dx} e^{3x}\cos(5x) = e^{3x}\cdot 3\cos(5x) + e^{3x}\cdot(-\sin(5x))\cdot 5$

$= e^{3x}(3\cos(5x) - 5\sin(5x))$

3. (8pts) $\frac{d}{dx}\frac{3x-1}{x^2+x-3} = \frac{3(x^2+x-3)-(3x-1)(2x+1)}{(x^2+x-3)^2} = \frac{3x^2+3x-9-(6x^2+x-1)}{(x^2+x-3)^2}$

$= \frac{-3x^2+2x-8}{(x^2+x-3)^2}$

4. (7pts) $\frac{d}{dx}\arctan(\sqrt{x}) = \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$

5. (8pts) $\frac{d}{dx}\ln(\tan(x^4+x^2+1))) = \frac{1}{\tan(x^4+x^2+1)}\cdot\sec^2(x^4+x^2+1)\cdot(4x^3+2x)$

$= \frac{\cos(x^4+x^2+1)}{\sin(x^4+x^2+1)}\cdot\frac{1}{\cos^2(x^4+x^2+1)}\cdot 4x^3+2x = \frac{4x^3+2x}{\sin(x^4+x^2+1)\cos(x^4+x^2+1)}$

6. (10pts) Use logarithmic differentiation to find $\frac{d}{dx}(\sin x)^{\cos x}$.

$y = (\sin x)^{\cos x} \quad | \ln$

$\ln y = \cos x \ln(\sin x) \quad | \frac{d}{dx}$

$\frac{1}{y} \cdot y' = -\sin x \ln(\sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x \quad | \cdot y$

$y' = (\sin x)^{\cos x}\left(\frac{\cos^2 x}{\sin x} - \sin x \ln(\sin x)\right)$

7. (14pts) Use implicit differentiation to find the equation of the tangent line to the curve $x^2 + 2xy - y^2 + x = 2$ at $(1, 2)$.

$x^2 + 2xy - y^2 + x = 2 \quad | \frac{d}{dx}$

$2x + 2(1 \cdot y + xy') - 2yy' + 1 = 0$

$2xy' - 2yy' = -1 - 2y - 2x$

$y'(2x - 2y) = -1 - 2y - 2x$

$y' = \frac{-1 - 2y - 2x}{2x - 2y}$

when $x = 1, y = 2$

$y' = \frac{-1 - 2 \cdot 2 - 2 \cdot 1}{2 \cdot 1 - 2 \cdot 2} = \frac{-7}{-2} = \frac{7}{2}$

$y - 2 = \frac{7}{2}(x - 1)$

$y = \frac{7}{2}x - \frac{7}{2} + 2$

$y = \frac{7}{2}x - \frac{3}{2}$

8. (10pts) Determine the following higher derivatives:

$D^{199}e^{3x} =$ $3^{199} e^{3x}$

$y = e^{3x}$ $\quad y''' = e^{3x} \cdot 3 \cdot 3 \cdot 3$

$y' = e^{3x} \cdot 3$ $\quad$ etc.

$y'' = e^{3x} \cdot 3 \cdot 3$

$D^{58}\cos(5x) =$ $-5^{58} \cos(5x)$

$58 \div 4 = 14$, rem. 2

$(\cos x)'' = (-\sin x)' = -\cos x$

Derivatives of cos cycle, every time an extra factor 5 comes out,

9. (14pts) The position of a ball thrown upward with initial velocity 40 meters per second is given by $s(t) = -5t^2 + 40t$.
a) When does the ball reach its highest point and what is its altitude then?
b) When is the ball at height 60 meters? What is its velocity then?

a) Reaches highest point when $v(t) = 0$

$v(t) = -10t + 40$

$-10t + 40 = 0$

$t = 4$

$s(4) = -5 \cdot 4^2 + 40 \cdot 4$

$= -80 + 160$

$= 80$

b) $s(t) = 60$

$-5t^2 + 40t = 60$

$5t^2 - 40t + 60 = 0$

$t^2 - 8t + 12 = 0$

$(t-2)(t-6) = 0$

$t = 2, 6$

$v(2) = 20$ (on way up)

$v(6) = -20$ (on way down)