

1. (6pts) Suppose 80 votes are cast in an election among four candidates. After the first 45 votes are counted, the tallies are as follows: Nguyen 17, Hagarian 13, Xiang 8 and Perron 7.

- a) What is the minimal number of remaining votes Nguyen needs to be assured of a win?  
 b) What is the minimal number of remaining votes Xiang needs to be assured of a win?

N 17  
 H 13  
 X 8  
 P 7

a)  $80 - 45 = 35$  remain

Nguyen's biggest threat is from Hagarian, if H received 4 votes to tie with N, of the remaining 31 votes, N would need  $\frac{31}{2} = 15.5$ , i.e. **16 votes** to assure a win.

b) X needs 9 votes to catch up. Once caught up, of the remaining 26 votes, X needs  $\frac{26}{2} + 1 = 14$  votes to win. Thus, X needs  $9 + 14 = \mathbf{23}$  votes.

2. (4pts) If 165 votes are cast, what is the smallest number of votes a winning candidate can have in a 3-candidate race decided by plurality?

One wins with fewest votes if all candidates have nearly the same number of votes.

$\frac{165}{3} = 55$ . A candidate can win with  $55 + 1 = \mathbf{56}$  votes.

(For example 56, 54, 55)

3. (20pts) An ex-convicts' society is deciding on the primary object to appear in the society's logo. The preference rankings of the voters are shown below.

percentage of voters:	8	20	18	23	15	16
Barbed wire	1	1	2	3	2	3
Bars	2	3	1	1	3	2
Black sheep	3	2	3	2	1	1

- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with a runoff?
- Which choice is the Condorcet winner, if any?
- Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.
- In the Borda election, could the 15% of voters from the fifth column achieve that their second choice wins by voting strategically, assuming all the other members voted as shown?

a) wire 28      b) runoff  
 bars 41 wins  $\rightarrow 41 + 8 = 49$   
 sheep 31  $\rightarrow 31 + 20 = 51$  wins

c) wire  $28 + 15 = 43$   
 bars  $41 + 16 = 57$  wins

wire  $28 + 18 = 46$   
 sheep  $31 + 23 = 54$  wins

bars  $41 + 8 = 49$   
 sheep  $31 + 20 = 51$  wins

Sheep is the Condorcet winner

d) wire  $28 \cdot 3 + 33 \cdot 2 + 39 \cdot 1 = 189$   
 bars  $41 \cdot 3 + 24 \cdot 2 + 35 \cdot 1 = 206$  wins  
 sheep  $31 \cdot 3 + 43 \cdot 2 + 26 \cdot 1 = 205$

sum needs to be  $100 \cdot 6$

$\rightarrow 600$   
correct

e) Suppose the 15% voted

as wire 1  
 bars 3  
 sheep 2

subtract contribution of 15%  
 $\downarrow$

add altered contribution of 15%  
 $\downarrow$

$-15 \cdot 2 = 159$	$+15 \cdot 3 = 204$
$-15 \cdot 1 = 191$	$+15 \cdot 1 = 206$ wins
$-15 \cdot 3 = 160$	$+15 \cdot 2 = 190$

e) the 15% cannot achieve that "wire" wins.