Spring '08/MAT 117/Worksheet 5 **Name:** Solution *Show all your work.*

1. (6pts) Our experiment involves a "deck" of four cards: king, queen, jack and ace, all of hearts. We consecutively draw two cards from the deck (without returning).
a) List all the outcomes of this experiment. How many outcomes does this experiment have?
b) List the outcomes for which one of the cards is an ace.
c) What is the probability of drawing an ace among the two cards?

a) KA QA JA AJ
KJ QJ JQ AQ
KQ QK JK AK
12 outcomes

b) KA, QA, JA,
AJ, AQ, AK

c) $P(\text{ace is one of cards}) = \frac{6}{12} = \frac{1}{2}$

2. (3pts) Suppose the house odds on the horse Faster Than Thou are 2 to 5. If you think the horse's chances of winning are 60%, is this a fair bet? Who does it favor?

$$\text{true odds against} = \frac{1-0.6}{0.6} = \frac{0.4}{0.6} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{2}{5} < \frac{2}{3}$$

house odds < true odds against

so unfair bet, favoring house.

3. (5pts) In a class of 15 students, 9 are wearing jeans, 7 are wearing a T-shirt with a statement, and 3 are wearing both. If a student is randomly selected, what is the probability
a) they are wearing jeans or a T-shirt with a statement?
b) they are not wearing at least one of the two clothing items?

a) $P(\text{jeans or T-shirt}) = P(\text{jeans}) + P(\text{T-shirt}) - P(\text{jeans and T-shirt})$

$$= \frac{9}{15} + \frac{7}{15} - \frac{3}{15} = \frac{13}{15}$$

b) $P(\text{not wearing at least one}) = P(\text{NOT}(\text{jeans and T-shirt}))$

$$= 1 - P(\text{jeans and T-shirt})$$

$$= 1 - \frac{3}{15} = \frac{12}{15}$$

4. (2pts) If a die is rolled, the odds against getting an even number are 3 to 3.
or 1 to 1

5. (9pts) The probability of winning a bet on a single number in roulette is 1/38. Suppose you play roulette two times in a row. What is the probability that
a) you win both times? b) you lose both times? c) you win exactly once?
d) Add your answers in a)–c). Is it what you expect? Why?

a) $P(\text{win both times}) = P(\text{1st win and 2nd win}) = P(\text{1st win}) \cdot P(\text{2nd win})$
$= \frac{1}{38} \cdot \frac{1}{38} = \frac{1}{1444}$

b) $P(\text{lose both times}) = P(\text{1st lose and 2nd lose}) = P(\text{1st lose}) \cdot P(\text{2nd lose})$
$= \frac{37}{38} \cdot \frac{37}{38} = \frac{1369}{1444}$

c) $P(\text{win exactly once}) = P((\text{1st win and 2nd lose}) \text{ or } (\text{1st lose and 2nd win}))$
$= P(\text{1st win})P(\text{2nd lose}) + P(\text{1st lose})P(\text{2nd win})$
$= \frac{1}{38} \cdot \frac{37}{38} + \frac{37}{38} \cdot \frac{1}{38} = \frac{74}{1444}$

d) $\frac{1+1369+74}{1444} = \frac{1444}{1444} = 1$ ← a), b), c) describe all the possibilities

$P(a) \text{ or } b) \text{ or } c)) = 1$

6. (5pts) A game of chance, with house odds 1 to 6 is set up as follows: you roll two dice and win if 5 or 10 is the sum on the dice. It costs 25 cents to play.
a) What is the expected gain or loss on one play of this game?
b) If you play 40 times, how much do you expect to gain or lose overall?

house odds 1 to 6 mean:
for every \$1, win \$1/6 (and \$1 is returned)

sum = 5 : 1,4 2,3 3,2 4,1
sum = 10 : 4,6 5,5 6,4
7 outcomes

a)

outcome	prob.
$0.25 \cdot \frac{1}{6}$	$P(\text{win}) = \frac{7}{36}$
-0.25	$P(\text{lose}) = \frac{29}{36}$

expected value $= 0.25 \cdot \frac{1}{6} \cdot \frac{7}{36} + (-0.25) \cdot \frac{29}{36}$
$= -0.193287$
lose about 19c per play

b) $40 \cdot 0.193287 = \$7.73$
lose about \$7.73
on 40 games