

$$\text{angle} = (\text{relative frequency}) \cdot 360^\circ \quad Z = \frac{X - \mu}{\sigma}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \quad \sigma = \sqrt{\frac{f_1(x_1 - \mu)^2 + f_2(x_2 - \mu)^2 + \cdots + f_n(x_n - \mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

$$\frac{a}{b} = \frac{1-P(E)}{P(E)} \quad P(E) = \frac{b}{a+b} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$F = P(1+rt) \quad F = P \left(1 + \frac{r}{n}\right)^{nt} \quad F = D \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = R \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad APY = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (4pts) If 99 votes are cast, what is the smallest number of votes a winning candidate can have in a three-candidate race that is decided by plurality? Justify your answer.

2. (5pts) Compute the following probability for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

$$P(.34 \leq Z \leq 1.19) =$$

3. (13pts) A group of opera critics are choosing their favorite present-day soprano. The preference rankings for three sopranos are below:

Number of votes:	3	7	4	3	5	4
Anna Netrebko	1	1	2	3	2	3
Angela Gheorghiu	2	3	1	1	3	2
Renée Fleming	3	2	3	2	1	1

- a) Which soprano wins using the plurality method?
- b) Which soprano wins using the Plurality with runoff method?
- c) Which soprano wins using the Borda method?
- d) Perform the check on the sum of Borda points.
- e) In the Borda method, can the three critics who voted Gheorghiu first and Fleming second obtain a preferable outcome if they voted strategically?

4. (12pts) There will be 67 total solar eclipses on Earth from 2001 to 2100. The duration of solar eclipses varies with the frequency distribution of the durations shown below.

- a) Find the median duration.
- b) Find the mean duration.
- c) Find the standard deviation.

Duration (min)	Frequency
2	19
3	19
4	14
5	8
6	7

5. (7pts)

- a) If one card is drawn from a deck of cards, what is the probability that it is an ace?
- b) If two cards are drawn from a deck of cards, what is the probability that the second card is a spade, given that the first card was a diamond?
- c) If two cards are drawn from a deck of cards, what is the probability that both are diamonds?

6. (8pts) A spinner can stop in one of four equal-sized fields labeled A, B, C, D. Using the spinner, you play the following game, for which there is a \$1 charge. If you get a B on a spin, you win \$3.00 (and your \$1 is returned), otherwise, you win nothing.

a) What is the expected value for this game of chance?

b) If you play the game 10 times, how much do you expect to win (or lose)?

c) What are the house odds on this bet? What are the true odds against the spinner stopping on B?

d) Use odds to determine whether this game represents a fair bet.

7. (4pts) Sam will take two final exams on the same day, one in psychology, one in biology. The probability there is a surprise question on the psychology exam is 15%, and the probability there is a surprise question on the biology exam is 20%. Assuming the writers of the two exams work independently, what is the probability that Sam gets a surprise question on both exams that day?

8. (4pts) If \$2,000 is deposited into an account bearing 4.53%, compounded monthly, how much is in the account after two-and-a-half years?

9. (5pts) Melissa would like to save money to buy a car for \$17,000. How much should she deposit every week into an account bearing 5%, compounded weekly, in order to save up for the car in three years?

10. (8pts) Angelina Jolie is building an orphanage in Vietnam for which she needs to borrow \$1,500,000. Suppose she can get a 15-year loan with interest rate 5.58%, compounded monthly.

- a) What is her monthly payment?
- b) What is the balance on the loan after 5 years?

Bonus. (7pts) A 1991 survey done by the U.S. Bureau of Justice shows that the age of inmates in state prisons was approximately normally distributed with mean 32.4 years and standard deviation 9.9 years. What is the percentage of inmates between the ages of 30 and 35? (Draw a picture of the normal distribution.)