

1. (5pts) The members of a marching band are voting to choose what time they want to hold their evening rehearsals. The results of an approval election are in the table. Which option wins using the approval method?

Number of votes:	28	33	40	19	9	24
6PM	X			X	X	
7PM		X		X		X
8PM			X		X	X

2. (6pts) Suppose there are 75 votes cast in an election between three candidates, decided by plurality. After the first 50 votes are counted, the tally is Xie 14, Vasilev 16, Bergthold 20.
- a) What is the minimal number of remaining votes Bergthold needs to be assured of a win?
- b) What is the minimal number of remaining votes Xie needs to be assured of a win?
- Justify your answers.

3. (14pts) Fans of a circus clown act are voting for their favorite clown. The preference rankings are as follows:

Number of votes:	11	12	7	20	14
Clippy	1	1	2	3	4
Snippy	2	3	1	4	2
Trippy	4	2	4	1	3
Flippy	3	4	3	2	1

- Which clown wins using the plurality method?
- Which clown wins using the plurality method, followed by a runoff of the two top finishers?
- In the plurality with runoff election, can the seven fans who ranked Snippy first obtain a preferable outcome if they voted strategically, assuming all the other fans voted as shown in the table?

4. (5pts) If 124 votes are cast, what is the smallest number of votes a winning candidate can have in a five-candidate race that is decided by plurality? Justify your answer.

5. (15pts) A group of students are devising a prank to play on a professor. The options are: steal couch from professor's office, then send photos of couch in various places around town¹; attach unflattering photos of professor to their car in a place where the professor is unlikely to see it, but others will; and break into professor's office and leave a dozen day-old chicks on their desk. The preference rankings for the prank are below:

Number of votes:	2	3	3	2	5
Couch kidnap	1	1	2	3	2
Photos on car	2	3	1	1	3
Day-old chicks	3	2	3	2	1

- a) Which prank is the Condorcet winner, if any?
- b) Which prank wins using the Borda method?
- c) Perform the check on the sum of Borda points.
- d) Can the two students that voted "couch kidnap" the last obtain a preferable outcome if they voted strategically using the Borda method?

¹actually performed on a math professor

6. (5pts) Suppose three candidates are running in an election decided by plurality with a runoff between the two top finishers. If the results of the first ballot are Lewis 135, Junghenn 115, Albert 143, what percentage of Junghenn supporters need to vote for Lewis in order for Lewis to win the election?

Bonus. (5pts) Devise a scenario with three candidates that shows that plurality with runoff does not satisfy the property of independence from irrelevant alternatives. (Recall that the property says: if A wins over B in a two-candidate race, then in a race with any additional candidates, B cannot win.) Your answer should be a table with preference rankings for the three candidates, along with a tally of votes showing the property above is violated.