Spring '08/MAT 117/Final Exam **Name:** Solution *Show all your work.*

angle = (relative frequency) $\cdot$ 360° $\quad Z = \dfrac{X-\mu}{\sigma}$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad \sigma = \sqrt{\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \qquad \sigma = \sqrt{\frac{f_1(x_1-\mu)^2 + f_2(x_2-\mu)^2 + \cdots + f_n(x_n-\mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

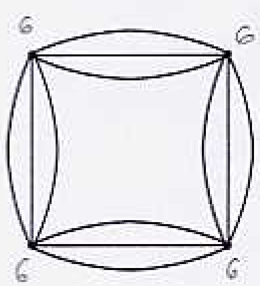
$\frac{a}{b} = \frac{1-P(E)}{P(E)} \quad P(E) = \frac{b}{a+b} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$F = P(1+rt) \quad F = P\left(1+\frac{r}{n}\right)^{nt} \quad F = D\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}} \quad P = R\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad APY = \left(1+\frac{r}{n}\right)^{n}-1$

1. (8pts) Determine whether the following graph has an Eulerian path or an Eulerian circuit. If it does, find it, if not, explain why not.

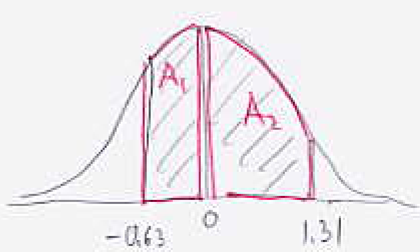


All vertices are even, so graph has an Eulerian circuit. Since a circuit is a path, it also has an Eulerian path.

2. (10pts) Compute the following probability for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

$P(-0.63 \leq Z < 1.31) =$ $A_1 + A_2 = 0.2357 + 0.4049$

$= \boxed{0.6406}$



3. (27pts) A group of film critics are choosing their favorite recent foreign film. The preference rankings for three candidates are below:

Percent of votes:	11	25	30	6	11	17
4 months, 3 weeks & 2 days,	1	1	2	3	2	3
Alexandra	2	3	1	1	3	2
Persepolis	3	2	3	2	1	1

a) Which film wins using the plurality method?
b) Which film wins using the Plurality with runoff method?
c) Which film wins using the Borda method?
d) Perform the check on the sum of Borda points.
e) In the Borda method, can the 17% of voters from the last column obtain a preferable outcome if they voted strategically?

a) 4 36 > tie between these
A 36
P 28

b) 4 36 + 11 = 47
A 36 + 17 = 53 wins

c)
4: 36·3 + 41·2 + 23·1 = 213 wins.
A: 36·3 + 28·2 + 36·1 = 200
P: 28·3 + 31·2 + 41·1 = 187

600

d) 100 · 6 = 600
each voter awards 6 pts

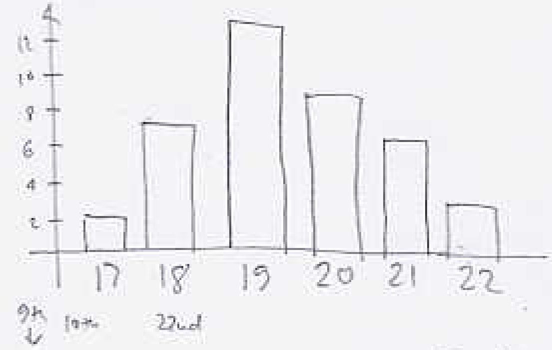
e) − contribution of the 17% | | + contribution if they voted 3 1 2
− 17·1 | = 196 | + 17·1 = 213
− 17·2 | = 166 | + 17·3 = 217 — wins
− 17·3 | = 136 | + 17·2 = 170

yes, they could cause "Alexandra" to win.

4. (23pts) The age distribution of a class is shown in the table.
a) Draw a histogram for the data.
b) Find the median age.
c) Find the mean age.
d) Find the standard deviation.

Age	Frequency
17	2
18	7
19	13
20	9
21	6
22	3
	40

a)



b) $17, 17, \underbrace{18, \ldots, 18}_{7}, \underbrace{19, \ldots, 19}_{13}, 20, \ldots, 20, 21, \ldots, 21, 22, \ldots, 22$

Need: 20th: 19 median age is $\frac{19+19}{2} = \boxed{19}$
21st: 19

c) $\mu = \dfrac{2\cdot17 + 7\cdot18 + 13\cdot19 + 9\cdot20 + 6\cdot21 + 3\cdot22}{40}$

$= \dfrac{779}{40} = \boxed{19.475}$

d) $\sigma^2 = \dfrac{2(17-\mu)^2 + 7(18-\mu)^2 + \ldots + 3(22-\mu)^2}{40}$

$\sigma^2 = \dfrac{65.975}{40} = 1.649375$

$\sigma = \sqrt{1.649375} = \boxed{1.284280}$

5. (13pts) A bag contains 7 red balls and 11 green ones.
a) If one ball is drawn from the bag, what is the probability that it is green?
b) If two balls are drawn from the bag, what is the probability that the second one is red, given that the first one was red?
c) If two balls are drawn from the bag, what is the probability that both are green?

a) $P(\text{one ball green}) = \boxed{\dfrac{11}{18}}$

b) $P(\text{2nd red} \mid \text{1st red}) = \boxed{\dfrac{6}{17}}$ ← red left, ← balls left

c) $P(\text{1st red and 2nd red}) = P(\text{1st red}) \cdot P(\text{2nd red} \mid \text{1st red})$

$= \dfrac{11}{18} \cdot \dfrac{10}{17} = \dfrac{11 \cdot \cancel{10}^{5}}{\cancel{18}_{9} \cdot 17} = \dfrac{55}{153} = 0.359478$

6. (17pts) A game of chance is set up as follows: you roll two dice and win \$4 if 3 or 7 is the sum on the dice. It costs \$1 to play (this \$1 is not returned when you win).
a) What is the probability of getting a sum of 3 or 7 on one roll of dice?
b) What is the expected gain or loss on one play of this game?
c) If you play 20 times, how much do you expect to gain or lose overall? expect to lose
d) Is this game a fair bet?

a) $P(\text{sum is } 3 \text{ or } 7) = \frac{8}{36} = \frac{2}{9}$

$3 = \begin{matrix} 2+1 \\ 1+2 \end{matrix} \quad 7 = \begin{matrix} 1+6 \\ 2+5 \\ 3+4 \\ 4+3 \\ 5+2 \\ 6+1 \end{matrix}$

c) $-20 \cdot 0.11111 = \boxed{-2.222222}$

d) Not a fair bet - favors the house (expected value is negative)

b) expected value $= 3 \cdot \frac{2}{9} + (-1)\frac{7}{9} = \frac{-1}{9} = -0.111111$

outcomes	P(outcome)
(4-1) $\rightarrow$ 3	$\frac{2}{9}$
-1	$\frac{7}{9}$

expect to lose about 11c

7. (8pts) In a group of 23 computers, 11 have Quicktime player installed, 9 have Winamp installed, and 5 have both media-playing programs installed. If a computer is randomly selected from the group, what is the probability that it has either of the programs installed?

$P(\text{Winamp or Quicktime}) = P(W) + P(Q) - P(W \text{ and } Q)$

$= \frac{9}{23} + \frac{11}{23} - \frac{5}{23}$

$= \boxed{\frac{15}{23}}$

8. (8pts) If \$11,000 is deposited into an account bearing 3.17%, compounded weekly, how much is in the account after three-and-a-quarter years?

$$F = P\left(1+\frac{r}{n}\right)^{nt}$$

$$F = 11{,}000\left(1+\frac{0.0317}{52}\right)^{52\cdot 3.25}$$

$$= 11{,}000\cdot(1.0006096\ldots)^{169}$$

$$= 11{,}000\cdot 1.108484$$

$$\boxed{= \$12193.33}$$

9. (10pts) When her daughter is born, Joanna decides to save \$250,000 to buy her a house or fund her college when she turns 18. How much should she deposit every quarter into an account bearing 9%, compounded quarterly?

$$F = D\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$250{,}000 = D\,\frac{\left(1+\frac{0.09}{4}\right)^{4\cdot 18}-1}{\frac{0.09}{4}}$$

$$250{,}000 = D\,\frac{1.0225^{72}-1}{0.0225}$$

$$250{,}000 = D\cdot 176.140\ldots$$

$$D = \frac{250{,}000}{176.140} = \boxed{\$1419.32 \text{ every quarter}}$$

10. (16pts) Britney Spears needs to borrow \$300,000 for a plastic operation that will change her appearance to the point where no one can recognize her. Suppose she can get a 20-year loan with interest rate 8%, compounded monthly.

a) What is her monthly payment?

b) What is the balance on the loan after 5 years?

a)

$$P = R\,\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$$

$$300{,}000 = R\cdot\frac{1-\left(1+\frac{0.08}{12}\right)^{-12\cdot 20}}{\frac{0.08}{12}}$$

$$300{,}000 = R\cdot\frac{1-(1.0066\ldots)^{-240}}{0.0066\ldots}$$

$$300{,}000 = R\cdot 119.55\ldots$$

$$R = \frac{300{,}000}{119.55} = \boxed{2509.32}$$

b) amount owed = present value of remaining payments

$$\text{owed} = 2509.32\cdot\frac{1-\left(1+\frac{0.08}{12}\right)^{-12\cdot 15}}{\frac{0.08}{12}}$$

$$= 2509.32\cdot 104.64\ldots$$

$$= \boxed{262{,}576.73}$$

Bonus. (14pts) In the late 1970s, the height of American women between 25 and 34 years of age was normally distributed with mean 64.5 inches and standard deviation 2 inches. Suppose we choose two women at random. What is the probability that the height of both of them is between 66 and 68 inches? (Assume their heights are independent of each other.)

$$Z = \frac{X-64.5}{2}$$

$$P(66 \le X \le 68) = P\left(\frac{66-64.5}{2} \le Z \le \frac{68-64.5}{2}\right)$$

$$= P(0.75 \le Z \le 1.75) = A_1 - A_2 = 0.4599 - 0.2734 = \boxed{0.1865}$$

P(both have heights between 66 and 68) = [independent events]

$$= P(\text{1st has this height})\cdot P(\text{2nd has this height}) = 0.1865\cdot 0.1865 = \boxed{0.0347823}$$