1. (23pts) An advertising agency has chosen the tagline "The world will end in _____!" as part of a campaign for a new cell phone. Copy writers voted on the last word in the tagline and the scores are below.

Number of votes:	4	3	1	3	2	4
"Fire"	1	1	2	3	2	3
"Ice"	2	3	1	1	3	2
"Desire"	3	2	3	2	1	1

> This problem brought to you by Robert Frost's **Fire and Ice**:
>
> Some say the world will end in fire,
> Some say in ice.
> From what I've tasted of desire
> I hold with those who favor fire.
> But if it had to perish twice,
> I think I know enough of hate
> To say that for destruction ice
> Is also great
> And would suffice.

a) Which word wins using the plurality method?
b) Which word wins using the plurality method, followed by a runoff of the two top finishers?
c) Which word is the Condercet winner, if any?

b) Runoff

a) F 7 ⟶ F 7+1 = 8
    I 4
    D 6 ⟶ D 6+3 = 9

"Fire" wins   "Desire" wins

c) Head-to head:

F . 7 + 2 = 9 w
I   4 + 4 = 8

F 7+1 = 8
D 6+3 = 9 wh

I 4+4 = 8
D 6+3 = 9 wins

"Desire" is the Condercet winner

2. (16pts) A PTA board must determine what kind of fundraiser it will hold. The choices, as well as approval votes are shown below.

percent of votes:	13	15	10	14	11	12	12	13
Holding a carneval	X			X	X		X	
Selling magazines						X	X	X
Selling wrapping paper		X		X		X	X	
Holding an auction				X		X		X

a) Which option wins using the approval method?
b) If half of the 14% of voters from the fourth column favor holding a carneval, could they have obtained a preferable result by voting strategically, if the others voted as shown?

a) Carneval $13 + 14 + 11 + 12 = 50$

Magazines $12 + 12 + 13 = 37$

Wrap paper $15 + 14 + 12 + 12 = 53$

Auction $10 + 11 + 13 = 34$

Selling wrapping paper wins.

b) That 7% group could have withdrawn their approval of "wrapping paper", thereby causing its count of 53 to drop by 7: $53 - 7 = 46$

Everything else stays same, so "Carneval" wins.

3. (12pts) Suppose there are 60 votes cast in an election between three candidates, decided by plurality. After the first 37 votes are counted, the tally is Wilson 15, Hoover 12, Coolidge 10.
a) What is the minimal number of remaining votes Wilson needs to be assured of a win?
b) What is the minimal number of remaining votes Coolidge needs to be assured of a win? Justify your answers.

23 votes remain

a) Hoover needs 3 votes to catch up with Wilson

Of remaining 20 votes, Wilson needs $\frac{20}{2} = 10$, $10 + 1 = 11$ votes to win.

⟨11 votes⟩

b) Coolidge needs 5 to catch up with Wilson. Of remaining 18 votes, he needs $\frac{18}{2} = 9$, $9 + 1 = 10$ votes

Overall, he needs $5 + 10 = $ ⟨15 votes⟩ to win.

4. (10pts) If 75 votes are cast, what is the smallest number of votes a winning candidate can have in a four-candidate race that is decided by plurality? Justify your answer.

A candidate wins with fewest votes
if scores of all candidates are nearly
the same.

$\frac{75}{4} = 18$, rem. 3

Of remaining 3 votes, a candidate
needs 2, to avoid a tie,

20, 19, 18, 18

5. (23pts) Members of an experimental church are deciding on a piece of music to begin their Sunday services with. The preference rankings for the choices are below:

Number of votes:	7	3	5	6	4	2
"Star Wars" theme	1	1	2	3	2	3
Madonna's "Like a Prayer"	2	3	1	1	3	2
Anthem of Panama	3	2	3	2	1	1

a) Which music wins using the Borda method?
b) Perform the check on the sum of Borda points.
c) Using the Borda method, could the four members from the fifth column obtain a preferable outcome by voting strategically, if the others voted as shown?

a) $10 \cdot 3 + 9 \cdot 2 + 8 \cdot 1 = 56$
   $11 \cdot 3 + 9 \cdot 2 + 7 \cdot 1 = 58$ wins
   $\underline{6 \cdot 3 + 9 \cdot 2 + 12 \cdot 1 = 48}$
   $\qquad\qquad 162$

b) 27 voters, each awards 6 pts
   $27 \cdot 6 = 162$

$\begin{vmatrix} -8 &= 48 \\ -4 &= 54 \\ -12 &= 36 \\ & 9 \end{vmatrix}$ $\begin{vmatrix} +12 &= 60 \text{ wins} \\ +4 &= 58 \\ +8 &= 44 \\ & \uparrow \end{vmatrix}$

Subtract   add contribution
contribution   if they ranked
of 5th column   options $\begin{matrix} 1 \\ 3 \\ 2 \end{matrix}$

c) yes they could have
caused "Star Wars" to win

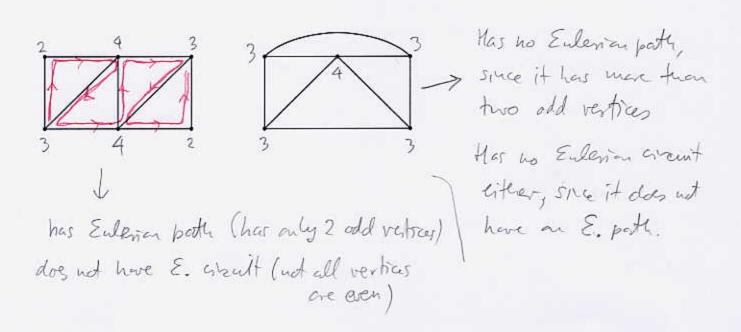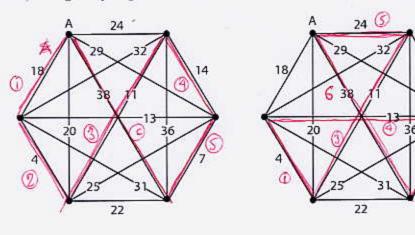**6.** (16pts) Determine whether each of the following graphs has an Eulerian path or an Eulerian circuit. If it does, find it, if not, explain why not.

Has no Eulerian path, since it has more than two odd vertices

Has no Eulerian circuit either, since it does not have an E. path.

has Eulerian path (has only 2 odd vertices)
does not have E. circuit (not all vertices are even)

Bonus. (10pts) Find an approximate solution to the traveling salesman problem for a circuit starting at point A. Show the weight of the found circuits. Use (one on each picture)
a) the nearest neighbor algorithm
b) the greedy algorithm.

$18 + 4 + 11 + 14 + 7 + 38$
$= 92 \leftarrow$ shorter

$4 + 11 + 24 + 38 + 7 + 13$
$= 97$