angle = (relative frequency) \cdot 360^\circ \\
Z = \frac{X - \mu}{\sigma}

\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \\
\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}

\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \\
\sigma = \sqrt{\frac{f_1(x_1 - \mu)^2 + f_2(x_2 - \mu)^2 + \cdots + f_n(x_n - \mu)^2}{f_1 + f_2 + \cdots + f_n}}

1. (16pts) A 1994 Consumer Reports survey found that the rates for a single room at a selection of budget motels were 42, 40, 40, 40, 40, 50, 38, 48, 45, 40, 32, 45, 49, 34.
   a) Find the mode of the data.
   b) Find the median rate.
   c) Find the mean rate.
   d) Find the standard deviation.

   a) 40 is the most frequent value

   b) 32, 34, 38, 40, 40, 40, 40, 42, 45, 45, 48, 49, 50  14 items

   \begin{align*}
   \text{median} &= \frac{40 + 40}{2} = 40
   \end{align*}

   c) \frac{32 + 34 + 38 + 5 \cdot 40 + 42 + 2 \cdot 45 + 48 + 49 + 50}{14} = \frac{583}{14} = 41.642857

   d) \sigma^2 = \frac{(32 - \mu)^2 + (34 - \mu)^2 + (38 - \mu)^2 + 5(40 - \mu)^2 + (42 - \mu)^2 + 2(45 - \mu)^2 + (48 - \mu)^2 + (49 - \mu)^2 + (50 - \mu)^2}{14}

   \begin{align*}
   &= \frac{365.21}{14} = 26.086 \\
   \sigma &= \sqrt{26.086} = 5.107517
2. (8pts) The median of a data set has the property that half the data is always below, and half is above the median. Give an example to show that the mean does not have this property. More precisely, give seven numbers so that five are below, and two are above the mean of the seven numbers. Compute the mean to verify.

For example: 2, 2, 3, 3, 3, 5, 7

\[
\frac{2 + 2 + 3 + 3 + 3 + 5 + 7}{7} = \frac{25}{7} = 3.571429 \quad \text{five below this}
\]
\[
2 \quad \text{above it}
\]

3. (23pts) The frequency distribution of scores on worksheet 3 of my two 117 classes are shown below.
a) Draw a histogram for the data.
b) Find the median of the scores.
c) Find the mean of the scores.
d) Find the standard deviation of the scores.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>16</td>
</tr>
<tr>
<td>29</td>
<td>8</td>
</tr>
<tr>
<td>28</td>
<td>4</td>
</tr>
<tr>
<td>27</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>23</td>
<td>4</td>
</tr>
<tr>
<td>18</td>
<td>4</td>
</tr>
</tbody>
</table>

\[\text{Total} = 41\]

\[\text{Median} = \frac{18 + 18}{2} = 18\]

\[\text{Mean} = \frac{30 \times 16 + 29 \times 8 + 28 \times 4 + 27 \times 2 + 24 \times 3 + 23 \times 4 + 18 \times 4}{41} = \frac{1114}{41} = 27.170732\]

\[\text{S}^2 = \frac{16(30-\mu)^2 + 8(29-\mu)^2 + 4(28-\mu)^2 + ... + 4(18-\mu)^2}{41} = \frac{593.80}{41} = 14.8588\]

\[\sigma = \sqrt{14.8588} = 3.805660\]
4. (19pts) The distribution of tuition rates for in-state students for US public colleges for 1993 is shown on the next page (those were the days!). Do the following:
   a) Find the relative frequencies.
   b) Draw a pie chart (find angles first).
   c) Enter a representative value for each interval.
   d) Estimate the mean of data.

<table>
<thead>
<tr>
<th>Range of Tuition</th>
<th>Number of Colleges</th>
<th>Relative Frequency</th>
<th>Angle</th>
<th>Representative Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000–5999</td>
<td>7</td>
<td>0.013333</td>
<td>5°</td>
<td>5500</td>
</tr>
<tr>
<td>4000–4999</td>
<td>31</td>
<td>0.059048</td>
<td>21°</td>
<td>1500</td>
</tr>
<tr>
<td>3000–3999</td>
<td>87</td>
<td>0.165714</td>
<td>60°</td>
<td>3500</td>
</tr>
<tr>
<td>2000–2999</td>
<td>181</td>
<td>0.344286</td>
<td>124°</td>
<td>2500</td>
</tr>
<tr>
<td>1000–1999</td>
<td>207</td>
<td>0.399428</td>
<td>142°</td>
<td>1500</td>
</tr>
<tr>
<td>0–999</td>
<td>12</td>
<td>0.022857</td>
<td>8°</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>525</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \mu = \frac{17 \times 55 + 31 \times 45 + 87 \times 35 + 181 \times 25 + 207 \times 15 + 12 \times 5}{525} \]

\[ \mu = \frac{12515}{525} = 23.83 \]

\[ \mu \approx \$2383.84 \]

5. (20pts) Compute the following probabilities for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

a) \( P(-0.4 \leq Z < -0.21) = A_1 - A_2 = 0.1554 - 0.0832 \]
   \[ = 0.0722 \]

b) \( P(Z > 0.33) = A_1 - A_2 = 0.5 - 0.1293 \]
   \[ = 0.3707 \]
6. (14pts) In the late 1970s, the height of American males between the ages of 25 and 34 was approximately normally distributed with mean 70 inches and standard deviation 2 inches. Find the percentage of men whose height was between 66 and 76 inches.

\[ P(66 \leq X \leq 76) = P\left(\frac{66-70}{2} \leq Z \leq \frac{76-70}{2}\right) = P(-2 \leq Z \leq 3) = A_1 + A_2 \]

\[ = 0.4772 + 0.4987 = 0.9759 \]

About 97.59%

Next, find the height.

**Bonus.** (10pts) a) For a standard normal distribution, find the 75-th percentile. (That is, find the number \( z_0 \) so that \( P(Z \leq z_0) = 0.75 \). Note that this is the reverse of our usual problem.) Draw a picture.

b) If scores on a test with many participants had a mean of 81 with standard deviation 9, which score will put you above 75% of the other test-takers? *Hint: use the answer from a).*

\[ \frac{X - 81}{9} = 0.67 \]

\[ X = 81 + 9 \times 0.67 \]

Needed to score 87.03 to be in 75-th percentile