Spring '08/MAT 117/Exam 3 **Name:** Solution *Show all your work.*

angle = (relative frequency) $\cdot$ 360° $\quad Z = \dfrac{X-\mu}{\sigma}$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad \sigma = \sqrt{\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \qquad \sigma = \sqrt{\frac{f_1(x_1-\mu)^2 + f_2(x_2-\mu)^2 + \cdots + f_n(x_n-\mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

1. (16pts) A 1994 Consumer Reports survey found that the rates for a single room at a selection of budget motels were 42, 40, 40, 40, 40, 50, 38, 48, 45, 40, 32, 45, 49, 34.
a) Find the mode of the data.
b) Find the median rate.
c) Find the mean rate.
d) Find the standard deviation.

a) 40 is the most frequent value

b) 32, 34, 38, 40, 40, 40, 40, 40, 42, 45, 45, 48, 49, 50 $\quad$ 14 items

7th 8th

median $= \dfrac{40+40}{2} = 40$

c) $\dfrac{32+34+38+5\cdot 40+42+2\cdot 45+48+49+50}{14} = \dfrac{583}{14} = 41.642857$

d) $\sigma^2 = \dfrac{(32-\mu)^2+(34-\mu)^2+(38-\mu)^2+5(40-\mu)^2+(42-\mu)^2+2(45-\mu)^2+(48-\mu)^2+(49-\mu)^2+(50-\mu)^2}{14}$

$= \dfrac{365.21\ldots}{14} = 26.086\ldots$

$\sigma = \sqrt{26.086\ldots} = 5.107517$

2. (8pts) The median of a data set has the property that half the data is always below, and half is above the median. Give an example to show that the mean does not have this property. More precisely, give seven numbers so that five are below, and two are above the mean of the seven numbers. Compute the mean to verify.

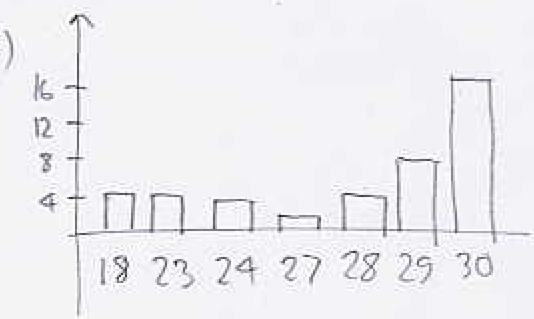
For example: 2, 2, 3, 3, 3, 5, 7

$$\frac{2\cdot 2+3\cdot 3+5+7}{7}=\frac{25}{7}=3.571429$$ ← five below this, 2 above it

3. (23pts) The frequency distribution of scores on worksheet 3 of my two 117 classes are shown below.
a) Draw a histogram for the data.
b) Find the median of the scores.
c) Find the mean of the scores.
d) Find the standard deviation of the scores.

Score	Frequency
30	16
29	8
28	4
27	2
24	3
23	4
18	4
	41

a)



17th 18th 25th

b) $\underbrace{18,\ldots,18}_{4}, \underbrace{23,\ldots,23}_{4}, \underbrace{24,\ldots,24}_{3}, \underbrace{27,27}_{2}, \underbrace{28,\ldots,28}_{4}, \underbrace{29,\ldots,29}_{8}, \underbrace{30,\ldots,30}_{16}$

$41/2=20.5$ need 21st on list, it is 8

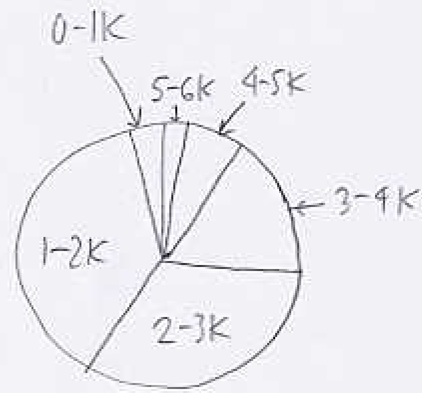
c) $$\mu=\frac{16\cdot 30+8\cdot 29+4\cdot 28+2\cdot 27+3\cdot 24+4\cdot 23+4\cdot 18}{41}=\frac{1114}{41}=27.170732$$

d) $$\sigma^2=\frac{16(30-\mu)^2+8(29-\mu)^2+4(28-\mu)^2+\ldots+4(18-\mu)^2}{41}=\frac{593.80\ldots}{41}=14.483\ldots$$

$$\sigma=\sqrt{14.483\ldots}=3.805660$$

4. (19pts) The distribution of tuition rates for in-state students for US public colleges for 1993 is shown on the next page (those were the days!). Do the following:
a) Find the relative frequencies.
b) Draw a pie chart (find angles first).
c) Enter a representative value for each interval.
d) Estimate the mean of data.

Range of Tuition	Number of Colleges	Relative Frequency	Angle	Representative Value
5000–5999	7	0.013333	5	5500
4000–4999	31	0.059048	21°	4500
3000–3999	87	0.165714	60°	3500
2000–2999	181	0.344762	124°	2500
1000–1999	207	0.394286	142°	1500
0–999	12	0.022857	8	500
	525			



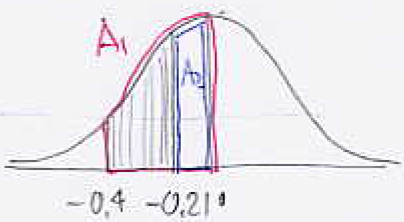
d) In hundreds: $\mu = \dfrac{7\cdot 55 + 31\cdot 45 + 87\cdot 35 + 181\cdot 25 + 207\cdot 15 + 12\cdot 5}{525}$

$= \dfrac{12515}{525} = 23.83\ldots$ $\quad \mu = \$2383.84$

5. (20pts) Compute the following probabilities for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

a) $P(-0.4 \le Z < -0.21) =$ $A_1 - A_2 = 0.1554 - 0.0832$

$= 0.0722$

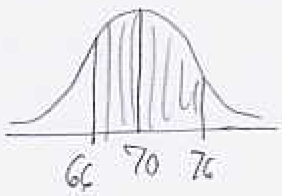


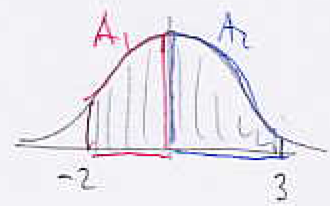
b) $P(Z > 0.33) =$ $A_1 - A_2 = 0.5 - 0.1293$

$= 0.3707$

6. (14pts) In the late 1970s, the height of American males between the ages of 25 and 34 was approximately normally distributed with mean 70 inches and standard deviation 2 inches. Find the percentage of men whose height was between 66 and 76 inches.

$$P(66 \le X \le 76) = P\left(\frac{66-70}{2} \le Z \le \frac{76-70}{2}\right) = P(-2 \le Z \le 3) = A_1 + A_2$$





$= 0.4772 + 0.4987$

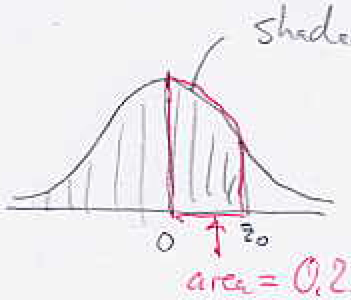
$= 0.9759$

About 97.59%

have this height

Bonus. (10pts) a) For a standard normal distribution, find the 75-th percentile. (That is, find the number z_0 so that $P(Z \le z_0) = 0.75$. Note that this is the reverse of our usual problem.) Draw a picture.
b) If scores on a test with many participants had a mean of 81 with standard deviation 9, which score will put you above 75% of the other test-takers? *Hint: use the answer from a).*

a)



closest to 0.25 in table are; 0.2486 ← closer
0.2517

0.2486 corresponds to 0.67

b)

$$\frac{x_0 - 81}{9} = 0.67$$

$$x_0 - 81 = 9 \cdot 0.67$$

$$x_0 = 81 + 9 \cdot 0.67$$

Needed to score 87.03 to be in 75th percentile