Spring '08/MAT 117/Exam 2 Name: Solution *Show all your work.*

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $P(E) = \frac{b}{a+b}$ $P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ or } B) = P(A)+P(B)-P(A \text{ and } B) = P(A)+P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

1. (4pts) If 11 students got an A on an exam in a class of 28, what is the probability that a random student picked from this class got less than an A on the exam?

$P(\text{less than } A) = P(\text{not } A) = 1 - P(A) = 1 - \frac{11}{28} = \frac{28-11}{28} = \frac{17}{28}$

2. (4pts) Jason walks to a park every day. In one month (30 days), he noticed that his shoelace got untied on 5 days. What is the experimental probability that Jason's shoelaces get untied on one trip to the park?

It is $\frac{5}{30} = \frac{1}{6}$

3. (14pts) A bag contains five gummi bears, one in each of colors: green, red, yellow, orange and purple. We draw two gummi bears consecutively from the bag (without returning).
a) List all the equally likely outcomes of this experiment. How many are there?
b) List the outcomes on which a red or a yellow gummi bear was drawn.
c) What is the probability of drawing a red or a yellow gummi bear?

a) G O P R Y

GO	OG	PG	RG	YG
GP	OP	PO	RO	YO
GR	OR	PR	RP	YP
GY	OY	PY	RY	YR

20 outcomes

b) GR OR PR RG YG
GY OY PY RO YO
RP YP
RY YR

14 outcomes

c) $P(\text{red or yellow}) = \frac{14}{20} = \frac{7}{10}$

4. (12pts) The horse Tail End won 8% of the races it took part in during the season so far. The house odds on it are 12 to 1 in the next race.
a) If you think its chances of winning are determined by its record so far, is this a fair bet, and whom does it favor?
b) If you bet \$25 on this horse, and it wins, how much will your bet return (in addition to your \$25)?

a) convert probability to odds:

$$\frac{a}{b} = \frac{1-P(E)}{P(E)} = \frac{1-0.08}{0.08} = \frac{0.92}{0.08} = \frac{92}{8} = \frac{23}{2}$$

house odds are $\frac{12}{1}$

$\frac{23}{2} < \frac{12}{1}$ so bet not fair, favors you

b) $25 \cdot \frac{12}{1} = 300$

win \$300 in addition to \$25.

5. (16pts) Suppose a multiple-choice exam has four possible answers for each question, only one of them correct. You get 5 points for each correct answer, lose 2 points for each incorrect answer and get nothing if you leave the question unanswered.
a) What is the expected point value of a random guess on this exam?
b) What is the expected point value if you can eliminate one of the answers as incorrect and choose a random answer from the remaining three?
c) Assuming you can always eliminate one answer and choose a random answer from the remaining ones, how many points would you expect to get on a test with 30 questions?

a)

outcome	probability
5	$P(\text{correct}) = \frac{1}{4}$
-2	$P(\text{incorrect}) = \frac{3}{4}$

expected value $= 5 \cdot \frac{1}{4} + (-2) \cdot \frac{3}{4}$

$= \frac{5-6}{4} = -\frac{1}{4}$

expect to lose $\frac{1}{4}$ pt per problem.

b)

outcome	probability
5	$P(\text{correct}) = \frac{1}{3}$
-2	$P(\text{incorrect}) = \frac{2}{3}$

expected value $= 5 \cdot \frac{1}{3} + (-2) \cdot \frac{2}{3}$

$= \frac{5-4}{3} = \frac{1}{3}$

expect to get $\frac{1}{3}$ pts per problem

c) $30 \cdot \frac{1}{3} = 10$ expect to get 10 points

6. (12pts) A fashion retailer is offering 42 pre-matched outfits on their website. 23 of the outfits have solid tops, 31 have solid bottoms and 39 have either tops or bottoms solid. If an outfit is randomly selected, what is the probability that
a) both its top and bottom is solid?
b) neither its top nor its bottom is solid?

a) P(top or bottom) = P(top) + P(bottom) − P(top and bottom)

$$\frac{39}{42} = \frac{23}{42} + \frac{31}{42} - P$$

$$\frac{39}{42} = \frac{54}{42} - P \quad \Big| + P$$

$$P + \frac{39}{42} = \frac{54}{42} \quad \Big| - \frac{39}{42}$$

$$P\begin{pmatrix}\text{top}\\ \text{and}\\ \text{bottom}\end{pmatrix} = \frac{15}{42}$$

b) P(neither top nor bottom)

= P(not (top or bottom))

= 1 − P(top or bottom)

$$= 1 - \frac{39}{42} = \frac{42-39}{42} = \frac{3}{42}$$

7. (20pts) A mail carrier encounters two mean dogs on his route. One of the dogs will chase him with 25% of the time, and the other will chase him with 12% of the time. Assume the two dogs act independently of each other. On a single run of the mail carrier's route, what is the probability:
a) that both dogs chase him?
b) that neither of the dogs chases him?
c) that at least one dog chases him?

independent

a) P(both dogs) = P(1st dog and 2nd dog) = P(1st dog) · P(2nd dog)
$= 0.25 \cdot 0.12 = 0.03$

b) P(neither dog) = P(not 1st dog and not 2nd dog)
= P(not 1st dog) · P(not 2nd dog) $= 0.75 \cdot 0.88 = 0.66$

c) P(at least one) = P(1st dog or 2nd dog)
= P(1st dog) + P(2nd dog) − P(1st dog and 2nd dog)
$= 0.25 + 0.12 - 0.03 = 0.34$

39 cars

8. (18pts) From a group of 15 American and 24 Japanese cars, two are chosen at random.
a) What is the probability that both cars are Japanese?
b) What is the probability that the second car is American, given that the first car was Japanese?
c) What is the probability that the second car is American?

a) $P(\text{both Jap.}) = P(\text{1st Jap. and 2nd Jap.})$

$= P(\text{1st Jap.}) \cdot P(\text{2nd Jap} \mid \text{1st Jap.}) = \frac{24}{39} \cdot \frac{23}{38} =$

b) $P(\text{2nd Amer.} \mid \text{1st Jap.}) = \frac{15}{38}$

c) $P(\text{2nd Amer.}) = \frac{15}{39}$ ← same as drawing just one

85

Bonus. (10pts) Chocolate lover Siu-Ling tracks her chocolate-eating habits during many weekends and finds that she has chocolate on a Saturday 65% of the time. If she has chocolate on a Saturday, then she has it on the following Sunday 35% of the time. If she does not have chocolate on a Saturday, then she has it on the following Sunday 85% of the time. What is the probability that she has chocolate on exactly one day during a random weekend?

$P(\text{exactly one day}) = P(\text{only Saturday or only Sunday})$

mutually exclusive

$P(\text{Saturday and not Sunday}) + P(\text{Sunday and not Saturday})$

$= P(\text{Saturday}) \cdot P(\text{not Sunday} \mid \text{Saturday}) + P(\text{not Saturday}) \cdot P(\text{Sunday} \mid \text{not Saturday})$

$= 0.65 \cdot 0.65 + 0.35 \cdot 0.85$

$= 0.4225 + 0.2975 = 0.7210$