1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

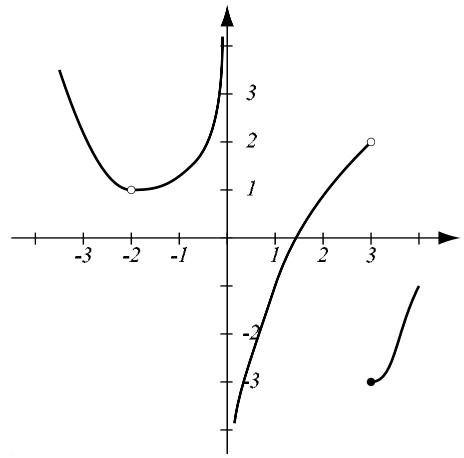
$\lim_{x\to 3^-} f(x) =$

$\lim_{x\to 3^+} f(x) =$

$\lim_{x\to 0} f(x) =$

$\lim_{x\to -2} f(x) =$

Is f continuous at $x = -2$ and why (not)?



2. (6pts) Find the following limits algebraically.

a) $\lim_{x\to 7} \dfrac{x^2 - 4x - 21}{x - 7} =$

b) $\lim_{x\to 3} \dfrac{5}{(x-3)^2} =$

3. (9pts) This problem is about the limit $\lim_{x\to 4} \dfrac{3x-12}{\sqrt{8x+1}-\sqrt{33}}$.

a) Use your calculator to estimate the limit with three accurate decimal places. Show the table of values.

b) Find the limit algebraically and compare your answer to a).

4. (5pts) Find $\lim_{x\to 0} (x^4+x^2)\sqrt{2+\sin\frac{1}{x}}$. Use the theorem that rhymes with what an allergy sufferer might do.

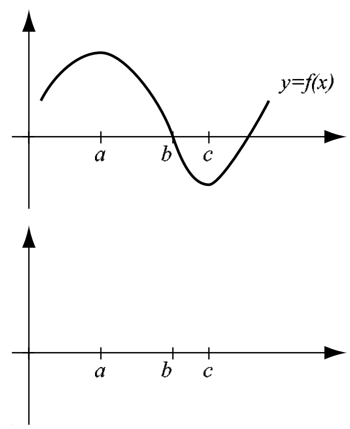
5. (4pts) Use the Intermediate Value Theorem to show that the equation $x^4 - x^3 + x - 17 = 0$ has at least one real solution.

6. (9pts) The position of a pear thrown upward with initial velocity 9 meters per second is given by $f(t) = 9t - 5t^2$.
a) Find the instantaneous velocity of the pear at time a.
b) At what time does the pear reach the biggest height, and what is that height?

7. (5pts) Is the function $f(x)$ continuous at $x = 2$? Explain.

$$f(x) = \begin{cases} 3x - 2, & \text{if } x \leq 2 \\ 12 - 4x, & \text{if } 2 < x. \end{cases}$$

8. (5pts) The graph of $f(x)$ is given. Estimate the numbers below and draw the graph of $f'(x)$ under the graph of $f(x)$.



$f'(a) =$

$f'(b) =$

$f'(c) =$

Bonus. (5pts) Algebraically find the limit of the exponential expression

$$\lim_{x \to \infty} 2^{\frac{x^3 + x + 1776}{x - x^2}} =$$

Spring '07/MAT 250/Exam 2 **Name:** *Show all your work.*

Differentiate and simplify where appropriate:

1. (4pts) $\dfrac{d}{dx}\left(7x^7 - \dfrac{1}{\sqrt[4]{x^3}} - \dfrac{7}{x^4} + e\right) =$

2. (4pts) $\dfrac{d}{dx}\, x^{10}e^{3x} =$

3. (4pts) $\dfrac{d}{dx}\,\dfrac{x^2+4}{3x-7} =$

4. (5pts) $\dfrac{d}{dx}\,\ln\left(\dfrac{2x+1}{3x-7}\right)^4 =$

5. (5pts) Use logarithmic differentiation to find $\dfrac{d}{dx}(x^2 + 3x - 1)^{\sin x}$.

6. (4pts) Find the equation of the tangent line to the curve $y = x^3 - 4x^2 + 7$ at the point $(1, 4)$.

7. (4pts) Find the first three derivatives of $f(x)$ and use them to find the formula for $f^{(n)}(x)$ if $f(x) = \ln x$.

8. (5pts) Use implicit differentiation to find y'.

$\tan(xy) = 3x^2 + 5y^4$

9. (8pts) A tank filled with 600 liters of water drains in 4 hours from an opening in the bottom. The volume of water in the tank after t hours is given by $V(t) = 600(1 - \frac{t}{4})^2$.
a) How much water is in the tank when $t = 2$?
b) At what rate is the water draining when $t = 2$? What are the units?
c) Interpret the meaning of the number in b) by approximating how much water there is in the tank at time $t = 2.1$.
d) What is the exact amount of water in the tank at time $t = 2.1$?

10. (7pts) A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks from the spotlight to the wall at a speed of 1.6 meters per second, how fast is the length of his shadow on the wall decreasing when he is 8 meters away from the spotlight?

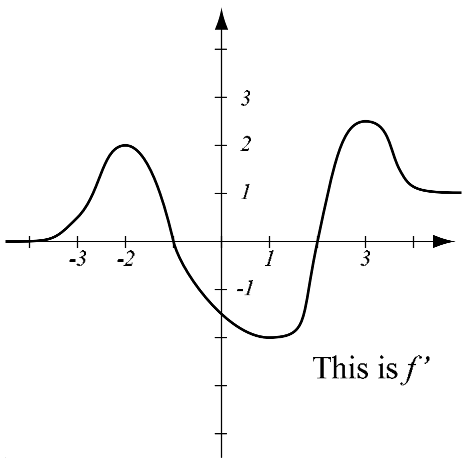
Bonus. (5pts) Let $h(x) = f(x)g(x)$. Find the formula for $h''(x)$ in terms of f, f', f'', g, g', g''. What familiar formula from algebra does it resemble?

1. (7pts) The graph of the function f' is given. Answer the following questions about f, which is defined for all real numbers (note: questions are not about f'). You may use a sign chart if it is helpful.

a) On which intervals is f increasing/decreasing?

b) On which intervals if f concave up/concave down?

c) At which points does f have local maxima/minima?



2. (10pts) Use L'Hospital's rule to find the limits:

a) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2} =$

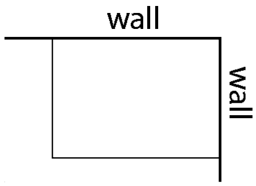
b) $\lim_{x\to 0^+} x^5 \ln x$

3. (7pts) Consider the function $f(x) = x^2 - 7x - 3$ on the interval $[2, 6]$.
a) Verify the hypotheses of the Mean Value Theorem.
b) Verify the conclusion of the Mean Value Theorem.

4. (7pts) Find the absolute minimum and maximum values for the function $f(x) = 4x - \tan x$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$.

5. (11pts) Let $f(x) = x^3 e^x$.
a) Find the intervals of increase/decrease and where f has a local maximum and minimum.
b) Find the intervals where f is concave up or down.
c) Use your calculator and the results of a) and b) to accurately sketch the graph of f.

6. (8pts) Sheila wishes to enclose a rectangular play pen so its area is 12m^2. Two sides of the pen are walls (see picture) and a fence is used for the remaining two sides. Find the dimensions of the pen that minimize the length of the fence. Show that the number you find, does, indeed, give you a minimum length.



Bonus. (5pts) Use information you gathered in problem 1 to draw the graph of the function f if it is known that f satisfies the additional conditions:

$f(0) = 4$

$\lim_{x \to -\infty} f(x) = 2$

What is $\lim_{x \to \infty} f(x)$?

1. (10pts) The function $f(x) = x^2 - 16$, $3 \leq x \leq 6$ is given.
a) Find the Riemann sum for the function with $n = 3$, taking sample points to be midpoints.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible.
c) What does the Riemann sum represent?

2. (2pts) Simplify using part 1 of the Fundamental Theorem of Calculus:

$$\frac{d}{dx} \int_2^x \frac{t^{14}}{t^6 + 4}\, dt =$$

3. (4pts) Write in sigma notation.

$$\frac{9}{5} + \frac{16}{6} + \frac{25}{7} + \frac{36}{8} + \frac{49}{9} =$$

4. (8pts) Find $\int_{-2}^{2}(x+1)\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Fundamental Theorem of Calculus.

5. (5pts) Find $f(x)$ if $f'(x) = x^3 - 3x$ and $f(2) = 5$.

Evaluate the following definite and indefinite integrals.

6. (4pts) $\displaystyle\int 5\sec^2 x - e^x\,dx =$

7. (6pts) $\displaystyle\int_1^3 \frac{x^4-1}{x^2}\,dx =$

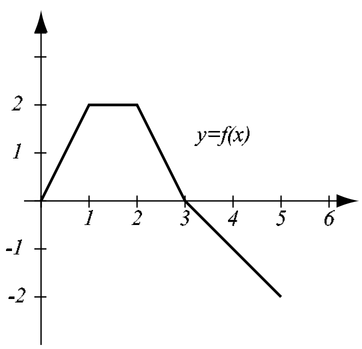
8. (6pts) Let $v(t) = \sqrt{t} - 1$ be the velocity of an inebriated snail (in millimeters per minute).
a) Calculate $\int_1^4 v(t)\,dt$ and state what it represents.
b) If the snail is 5mm away from a strawberry at time $t = 1$, and is moving away, what is its position at time $t = 4$?

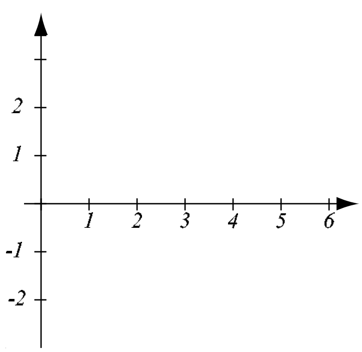
9. (5pts) Use the substitution rule to evaluate the indefinite integral.

$$\int \frac{\sin(\ln x)}{x}\, dx =$$

Bonus. (5pts) The graph of a function f is drawn below. Let $g(x) = \int_1^x f(t)\, dt$.
a) Fill in the table with values of g (note: NOT values of f).
b) Draw a nice graph of g, using values in the table and paying attention to where g is increasing/decreasing, concave up/down.

x	0	1	2	3	4
$g(x)$					





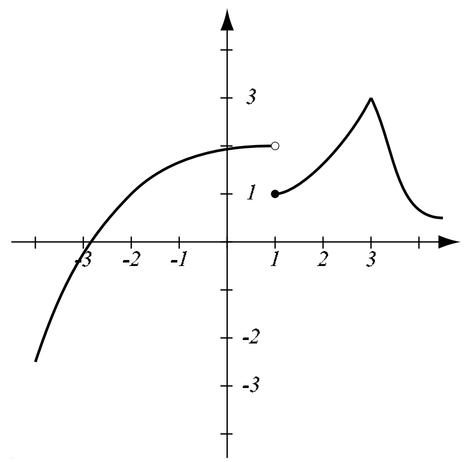
1. (6pts) Use the graph of the function to answer the following. Justify if a limit does not exist.

$\lim_{x \to -2} f(x) =$

$\lim_{x \to 1} f(x) =$

At which points is f discontinuous?

At which points is f not differentiable?



2. (6pts) Find the limit $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 + x - 12}$ in two ways:

a) algebraically

b) Using L'Hospital's rule

3. (5pts) Use implicit differentiation to find y'.

$x^2 + y^2 = x^2y^2$

4. (7pts) Let $P(t) = 150e^{0.04t}$ be the population (in thousands) of a certain city at time t, measured in years.

a) What is the population $t = 3$?

b) At what rate is the population increasing when $t = 3$? What are the units?

c) Interpret the meaning of the number in b) by approximating the population at time $t = 3.25$.

d) What is the exact population at time $t = 3.25$?

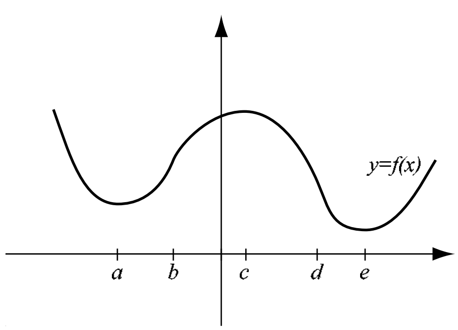
5. (9pts) Let $f(x) = \dfrac{1}{x^2+4}$.

a) Find the intervals of increase/decrease and where f has a local maximum and minimum.

b) Find the intervals where f is concave up or down.

c) Use your calculator and the results of a) and b) to accurately sketch the graph of f.

6. (5pts) The graph of f is shown in the picture. Draw the graphs of f' and f''.



7. (5pts) Let $f(x) = \sqrt{x}$. Find $f'(x)$ using the definition of the derivative. Then compare to what you get using a differentiation rule.

8. (7pts) The line $y = 8 - \frac{2}{3}x$ makes a triangle with the x- and y-axes. A rectangle may be inscribed in this triangle so that one vertex is on the line and two sides lie on the axes. Among all such rectangles, find the one that has the greatest area.

9. (6pts) Consider the integral $\int_0^{\frac{2\pi}{3}} \cos x \, dx$.
a) Use a picture to determine whether this definite integral is positive or negative.
b) Evaluate the integral and verify your conclusion from a).

10. (5pts) Use substitution (don't forget to change bounds) to evaluate:

$$\int_0^1 x^2(1+2x^3)^5 \, dx =$$

11. (5pts) Simplify using part 1 of the Fundamental Theorem of Calculus. (Hint: if $g(y) = \int_2^y t^2 \sin t \, dt$, you are trying to find the derivative of $g(y^2)$. What is $g'(y)$?)

$$\frac{d}{dy} \int_2^{y^2} t^2 \sin t \, dt =$$

12. (4pts) Use the Intermediate Value Theorem to show that the equation $e^x = x^2$ has a solution in $[-2, 2]$.

Bonus. (7pts) A camera 5km away from the launch pad tracks a rocket as it lifts off. As the rocket moves upwards, the camera keeps it in its view. When the angle with the horizontal of the camera is $\frac{\pi}{4}$ radians, the angle is increasing at rate 0.1 radians per second. What is the speed of the rocket at that moment?