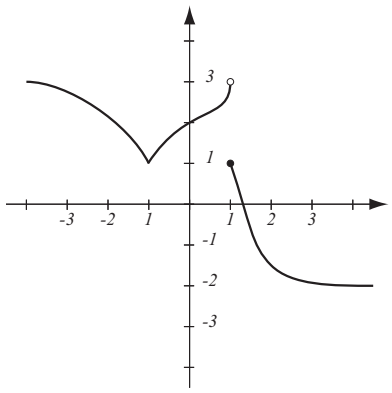
Spring '05/MAT 250/Final Exam **Name:** *Show all your work.*

1. (5pts) Use the graph of the function f at right to answer:
a) At which points x is f not continuous?
b) At which points x is f not differentiable?
c) $\lim_{x\to 1-} f(x) =$
d) $\lim_{x\to\infty} f(x) =$



Differentiate and simplify where appropriate:

2. (4pts) $\dfrac{d}{dx}\,(x^3 + 4x^2 + 1)e^{2x} =$

3. (3pts) $\dfrac{d}{dx}\,\sin(\ln x) =$

4. (3pts) The edge of a cube was found to be 30cm with a possible error in measurement of 0.1cm. Use differentials to estimate the maximum possible error in computing the volume of the cube.

5. (4pts) Find the limit algebraically. Do not use L'Hospital's rule.

a) $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9} =$

6. (4pts) Use L'Hospital's rule to find the limit:

$\lim_{x \to 0+} \dfrac{\sin x}{\sqrt{x}} =$

7. (5pts) Evaluate.

$\int_0^8 (e^{-x} + \sqrt[3]{x})\, dx =$

8. (4pts) Use substitution to find:

$\int \dfrac{\sec^2 x}{\tan x}\, dx =$

9. (8pts) Let $f(x) = xe^{x^2}$.

a) Find the intervals of increase/decrease and where f has a local maximum and minimum.

b) Find the intervals where f is concave up or down.

c) Use your calculator and the results of a) and b) to accurately sketch the graph of f.

10. (6pts) Use implicit differentiation to find y'.

$x \cos y = 4 + x^2 y^2$

11. (4pts) Use a graph to determine whether $\int_0^2 (x^2 - 1)\, dx$ is positive or negative. Then evaluate the integral and verify your answer.

12. (3pts) The velocity of a jet-ski is given by $v(t) = 3 + t$ meters per second. By how much did it change position from time $t = 2$ to $t = 4$?

13. (6pts) Use the closed interval method to find the absolute minimum and maximum values for the function $f(x) = 2x^3 + 3x^2 - 36x + 17$ on the interval $[0, 4]$.

14. (7pts) A kite is moving horizontally at altitude 40 meters and speed 2 meters per second. At what is the angle between the string and the horizontal decreasing when 80m of string have been let out?

15. (4pts) Use the Intermediate Value Theorem to show that the equation $x - \cos x = 0$ has a solution in the interval $[0, \pi/2]$.

Bonus 1. (4pts) Find a formula for the n-th derivative of $f(x) = \dfrac{1}{x^4}$.

Bonus 2. (4pts) A toll-road ticket shows a motorist entering a 160-mile long highway at 1:00PM and exiting at 3:00PM. If the speed limit on the road is 65mph, explain how a policeman armed with the Mean Value Theorem can prove that the motorist was speeding.