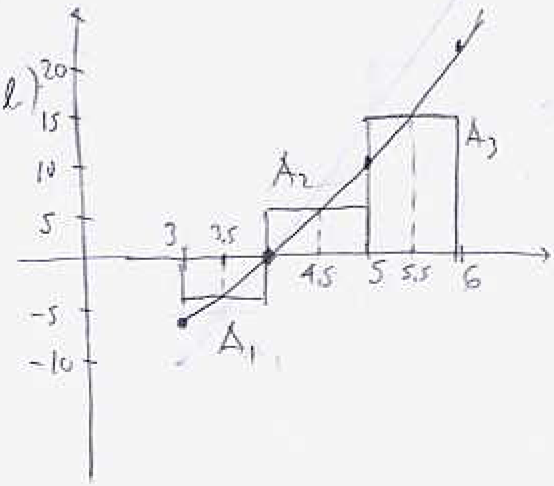
Spring '07/MAT 250/Exam 4 **Name:** Solution *Show all your work.*

1. (10pts) The function $f(x) = x^2 - 16$, $3 \le x \le 6$ is given.
a) Find the Riemann sum for the function with $n = 3$, taking sample points to be midpoints.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible.
c) What does the Riemann sum represent?

$\Delta x = 1$

a) $R_3 = \Delta x\left(f(3.5) + f(4.5) + f(5.5)\right)$
$= 1\cdot(3.5^2 - 16 + 4.5^2 - 16 + 5.5^2 - 16)$
$= 12.25 + 20.25 + 30.25 - 48$
$= 62.75 - 48 = 14.75$

b)



c) $R_3 = \underbrace{-A_1 + A_2 + A_3}_{\text{areas of rectangles}}$

2. (2pts) Simplify using part 1 of the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_2^x \frac{t^{14}}{t^6+4}\,dt = \frac{x^{14}}{x^6+4}$$

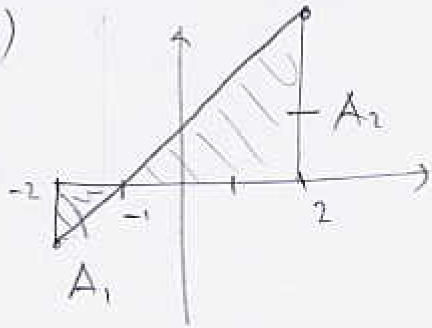
3. (4pts) Write in sigma notation.

$$\frac{9}{5} + \frac{16}{6} + \frac{25}{7} + \frac{36}{8} + \frac{49}{9} = \sum_{i=5}^{9} \frac{(i-2)^2}{i}$$

$$\text{or} = \sum_{i=3}^{7} \frac{i^2}{i+2}$$

4. (8pts) Find $\int_{-2}^{2}(x+1)\,dx$ in two ways:
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Fundamental Theorem of Calculus.

a)



$$\int_{-2}^{2}(x+1)\,dx = -A_1 + A_2$$

$$= -\frac{1}{2}\cdot 1\cdot 1 + \frac{1}{2}\,3\cdot 3$$

$$= -\frac{1}{2} + \frac{9}{2} = 4$$

b) $$\int_{-2}^{2} x+1\,dx = \left(\frac{x^2}{2}+x\right)\Big|_{-2}^{2} = \frac{2^2}{2}+2-\left(\frac{(-2)^2}{2}-2\right) = 4$$

5. (5pts) Find $f(x)$ if $f'(x) = x^3 - 3x$ and $f(2) = 5$.

$$f'(x) = x^3 - 3x$$

$$f(x) = \frac{x^4}{4} - 3\frac{x^2}{2} + C$$

$$f(x) = \frac{x^4}{4} - \frac{3x^2}{2} + 7$$

$$5 = f(2) = \frac{2^4}{4} - 3\cdot\frac{2^2}{2} + C$$

$$5 = 4 - 6 + C$$

$$7 = C$$

Evaluate the following definite and indefinite integrals.

6. (4pts) $\int 5\sec^2 x - e^x \, dx = 5\tan x - e^x + C$

7. (6pts) $\int_1^3 \frac{x^4 - 1}{x^2}\, dx = \int_1^3 \frac{x^4}{x^2} - \frac{1}{x^2}\, dx = \int_1^3 x^2 - x^{-2}\, dx$

$= \frac{x^3}{3} - \frac{x^{-1}}{-1}\Big|_1^3 = \left(\frac{x^3}{3} + \frac{1}{x}\right)\Big|_1^3 = \frac{3^3}{3} + \frac{1}{3} - \left(\frac{1^3}{3} + \frac{1}{1}\right)$

$= 9 + \frac{1}{3} - \left(\frac{1}{3} + 1\right) = 8$

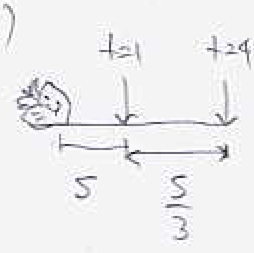
8. (6pts) Let $v(t) = \sqrt{t} - 1$ be the velocity of an inebriated snail (in millimeters per minute).
a) Calculate $\int_1^4 v(t)\, dt$ and state what it represents.
b) If the snail is 5mm away from a strawberry at time $t = 1$, and is moving away, what is its position at time $t = 4$?

a) $\int_1^4 \sqrt{t} - 1 \, dt = \int_1^4 t^{1/2} - 1\, dt = \left(\frac{t^{3/2}}{\frac{3}{2}} - t\right)\Big|_1^4$

$= \left(\frac{2}{3} t^{\frac{3}{2}} - t\right)\Big|_1^4 = \frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) - (4-1) =$

$= \frac{2}{3}(8-1) - 3 = \frac{14}{3} - 3 = \frac{5}{3}$ ← change in position from t=1 to t=4

b) at $t=4$ it is $5 + \frac{5}{3} = \frac{20}{3}$ mm away from strawberry



9. (5pts) Use the substitution rule to evaluate the indefinite integral.

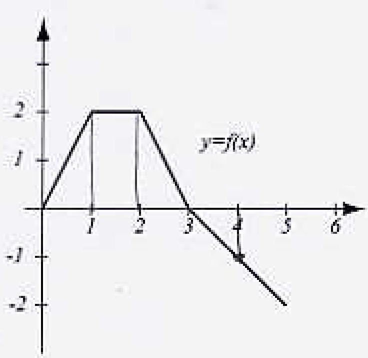
$$\int \frac{\sin(\ln x)}{x}\,dx = \left[\begin{array}{l} u=\ln x \\ du=\frac{1}{x}\,dx \end{array}\right] = \int \sin u\,du = -\cos u$$

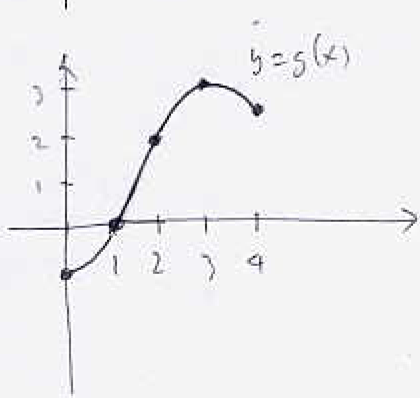
$\sin(\ln x)\cdot\frac{1}{x}$

$$= -\cos(\ln x)+C$$

Bonus. (5pts) The graph of a function f is drawn below. Let $g(x)=\int_1^x f(t)\,dt$.
a) Fill in the table with values of g (note: NOT values of f).
b) Draw a nice graph of g, using values in the table and paying attention to where g is increasing/decreasing, concave up/down.

x	0	1	2	3	4
$g(x)$	-1	0	2	3	2.5





$g(0)=\int_1^0 f = -\int_0^1 = -\frac{1}{2}\,1.2=-1$

$g(1)=\int_1^1 f=0$

$g(2)=\int_1^2 f=2$

$g(3)=\int_1^3 f=2+1=3$

$g(4)=\int_1^4 f=3-\frac{1}{2}\cdot 1\cdot 1=2{,}5$

g is incr. on $(0,3)$, decr. on $(3,4)$

$g'=f$ so g is CU on $(0,1)$, CD on $(2,4)$

(where f is incr, decr)