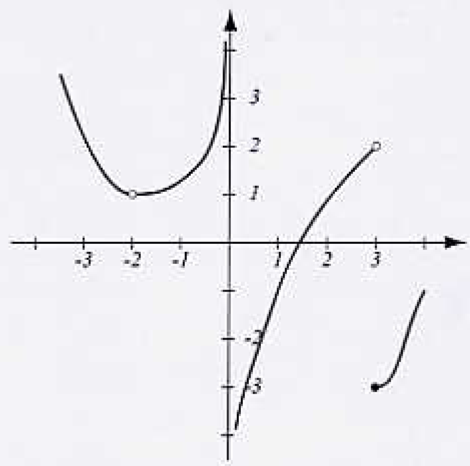
Spring '07/MAT 250/Exam 1 **Name:** Solution *Show all your work.*

1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.



$\lim_{x\to 3^-} f(x) =$ 2

$\lim_{x\to 3^+} f(x) =$ −3

$\lim_{x\to 0} f(x) =$ d.n.e., one-sided limits are different

$\lim_{x\to -2} f(x) =$ 1

Is f continuous at $x = -2$ and why (not)?

No, $f(-2)$ is not defined

2. (6pts) Find the following limits algebraically.

a) $\lim_{x\to 7} \frac{x^2-4x-21}{x-7} = \lim_{x\to 7} \frac{(x+3)\cancel{(x-7)}}{\cancel{x-7}} = \lim_{x\to 7} x+3 = 10$

b) $\lim_{x\to 3} \frac{5}{(x-3)^2} = \frac{5}{0+} = \infty$

$(x-3)^2 > 0$ and $(x-3)^2 \to 0$ when $x \to 3$

3. (~~4~~ 9pts) This problem is about the limit $\lim_{x\to 4} \frac{3x-12}{\sqrt{8x+1}-\sqrt{33}}$.

a) Use your calculator to estimate the limit with three accurate decimal places. Show the table of values.

b) Find the limit algebraically and compare your answer to a).

a)

x	$f(x)$
4.01	4.31103...
4.001	4.30868...
4.0001	4.30844...
4.00001	4.30842...

have stabilized

Limit appears to be about 4.308

b) $\lim_{x\to 4} \frac{3x-12}{\sqrt{8x+1}-\sqrt{33}} \cdot \frac{\sqrt{8x+1}+\sqrt{33}}{\sqrt{8x+1}+\sqrt{33}}$

$= \lim_{x\to 4} \frac{(3x-12)(\sqrt{8x+1}+\sqrt{33})}{(\sqrt{8x+1})^2-(\sqrt{33})^2}$

$= \lim_{x\to 4} \frac{(3x-12)(\sqrt{8x+1}+\sqrt{33})}{8x+1-33} \leftarrow 8x-32$

$= \lim_{x\to 4} \frac{3\cancel{(x-4)}(\sqrt{8x+1}+\sqrt{33})}{8\cancel{(x-4)}}$

$= \lim_{x\to 4} 3(\sqrt{8x+1}+\sqrt{33}) = \frac{3(\sqrt{33}+\sqrt{33})}{8}$

$= \frac{3\sqrt{33}}{4} \approx 4.308...$

4. (5pts) Find $\lim_{x\to 0}(x^4+x^2)\sqrt{2+\sin\frac{1}{x}}$. Use the theorem that rhymes with what an allergy sufferer might do.

$-1 \le \sin\frac{1}{x} \le 1 \quad |+2$

$1 \le 2+\sin\frac{1}{x} \le 3 \quad |\sqrt{\ }$

$1 \le \sqrt{2+\sin\frac{1}{x}} \le \sqrt{3} \quad |\cdot(x^4+x^2)$

$x^4+x^2 \le (x^4+x^2)\sqrt{2+\sin\frac{1}{x}} \le \sqrt{3}(x^4+x^2)$

$\lim_{x\to 0}(x^4+x^2) = 0+0 = 0$

$\lim_{x\to 0}\sqrt{3}(x^4+x^2) = \sqrt{3}(0+0) = 0$

By the squeeze theorem

$\lim_{x\to 0}(x^4+x^2)\sqrt{2+\sin\frac{1}{x}} = 0$

5. (4pts) Use the Intermediate Value Theorem to show that the equation $x^4 - x^3 + x - 17 = 0$ has at least one real solution.

Let $f(x) = x^4 - x^3 + x - 17$

$\rightarrow f(0) = -17$

$f(2) = 16 - 8 + 2 - 17 = -7$

$\rightarrow f(3) = 81 - 27 + 3 - 17 = 30$

Since 0 is between -17 and 30, by IVT there exists a number c in $(0,3)$ so that $f(c) = 0$.

6. (~~8~~ 9pts) The position of a pear thrown upward with initial velocity 9 meters per second is given by $f(t) = 9t - 5t^2$.

a) Find the instantaneous velocity of the pear at time a.

b) At what time does the pear reach the biggest height, and what is that height?

a) $$v(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{9(a+h) - 5(a+h)^2 - (9a - 5a^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{9a} + 9h - 5(\cancel{a^2} + 2ah + h^2) - \cancel{9a} + \cancel{5a^2}}{h}$$

$$= \lim_{h\to 0} \frac{9h - 10ah - 5h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(9 - 10a - 5h)}{\cancel{h}}$$

$$= 9 - 10a$$

b) $v(t) = 9 - 10t$

A biggest height when $v(t) = 0$

$9 - 10t = 0$

$t = \frac{9}{10} = 0.9\,s$

Biggest height:

$f(0.9) = 9 \cdot 0.9 - 5 \cdot 0.9^2$

$= 4.05\ m$

5

7. (4pts) Is the function $f(x)$ continuous at $x = 2$? Explain.

$$f(x) = \begin{cases} 3x - 2, & \text{if } x \le 2 \\ 12 - 4x, & \text{if } 2 < x. \end{cases}$$

$f(2) = 3 \cdot 2 - 2 = 4$

$\lim_{x \to 2} f(x) = 4$ since:

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 12 - 4x = 12 - 4 \cdot 2 = 4$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 3x - 2 = 6 - 2 = 4$

Since $\lim_{x \to 2} f(x) = 4 = f(2)$

function is continuous at $x = 2$

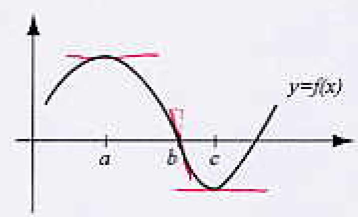
5

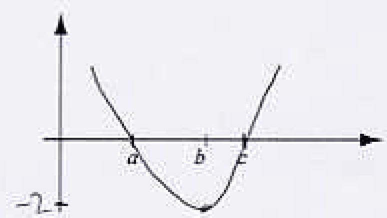
8. (4pts) The graph of $f(x)$ is given. Estimate the numbers below and draw the graph of $f'(x)$ under the graph of $f(x)$.

$f'(a) =$ 0

$f'(b) =$ −2

$f'(c) =$ 0

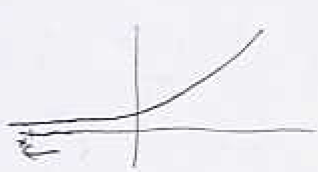




Bonus. (5pts) Algebraically find the limit of the exponential expression

$$\lim_{x \to \infty} 2^{\underbrace{\frac{x^3 + x + 1776}{x - x^2}}_{\to -\infty}} = 0$$

$$\lim_{x \to \infty} \frac{x^3 + x + 1776}{x - x^2} = \lim_{x \to \infty} \frac{x^3\left(1 + \frac{1}{x^2} + \frac{1776}{x^3}\right)}{x^2\left(\frac{1}{x} - 1\right)} = \lim_{x \to \infty} x \cdot \frac{1 + 0 + 0}{0 - 1} = \infty \cdot (-1) = -\infty$$



when $x \to -\infty$

$2^x \to 0$