

1. (6pts) Suppose 140 votes are cast in an election among five candidates. After the first 100 votes are counted, the tallies are as follows: Stein 12, O'Rourke 23, Cohen 17, Holt 29, Massey 19.

- a) What is the minimal number of remaining votes Holt needs to be assured of a win?  
 b) What is the minimal number of remaining votes Cohen needs to be assured of a win?

a) S 12  
 O' 23  
 C 17  
 H 29  
 M 19

In a worst case scenario for Holt  
 O'Rourke catches up with Holt  
 with 6 of remaining 40 votes.  
 Of the 34 that remain,  
 Holt needs 18 to secure a win.

1) Cohen needs 12 votes first  
 to catch up with Holt.

Then, of the remaining  $40 - 12 = 28$   
 votes C needs 15 to be  
 assured of a win.

Thus, Cohen needs  
 $12 + 15 = \underline{27}$  votes

2. (4pts) If 273 votes are cast, what is the smallest number of votes a winning candidate can have in a 4-candidate race decided by plurality?

$$273 \div 4 = 68, \text{ remainder } 1$$

If votes are equally distributed  
 among candidates the remaining 1 vote  
 will be enough to get someone to win.

So, 69 votes.

3. (20pts) A small town must decide whether to build tennis courts, a basketball court, or a baseball field. The residents are polled and their preference rankings are as follows:

percentage of voters:	12	20	11	24	10	23
Tennis courts	1	1	2	3	2	3
Basketball court	2	3	1	1	3	2
Baseball field	3	2	3	2	1	1

- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with a runoff?
- Which choice is the Condorcet winner, if any?
- Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.
- In the Borda election, could the 12% of voters from the first column achieve a preferable outcome by voting strategically, assuming all the other members voted as shown?

a) Tennis  $12+20=32$   
 Basket  $11+24=35 \leftarrow$  wins  
 Baseball  $10+23=33$

d) Tennis  $32 \cdot 3 + 21 \cdot 2 + 47 \cdot 1 = 185$   
 Basket  $35 \cdot 3 + 35 \cdot 2 + 30 \cdot 1 = 205$   
 Baseball  $33 \cdot 3 + 44 \cdot 2 + 23 \cdot 1 = 210$  wins  
 $\frac{600}{600} \checkmark$

b) Basketball, Baseball in runoff:

Basket:  $35 + 12 = 47$   
 Baseball:  $33 + 20 = 53 \leftarrow$  wins.

Borda pts should add  
 up to  $100 \cdot 6 = 60$

c) Tennis  $32 + 10 = 42$   
 Basket  $35 + 23 = 58$  wins

Tennis  $32 + 11 = 43$   
 Baseball  $33 + 24 = 57$  wins

Basket  $35 + 12 = 47$   
 Baseball  $33 + 20 = 53$  wins

Baseball is the Condorcet winner

e) Tennis  $185 \left| \begin{array}{l} -36 \\ -24 \\ -12 \end{array} \right| 149 \left| \begin{array}{l} +24 \\ +36 \\ +12 \end{array} \right| 173$   
 Basket  $205$   
 Baseball  $210$

contribution of the 12%  
 score w/o contribution of the 12%  
 contribution if the 12% change their vote to

T 2  
 B 1  
 B 3

Yes, they could cause basketball to win.