**1.** (6pts)  Two dice are rolled.
a) How many outcomes does this experiment have?
b) List the outcomes for which the sum on the dice is 8.
c) What is the probability of getting a sum of 8 on one roll of two dice?

a) 36 outcomes

$1,1 \quad 2,1 \quad \cdots \quad 6,1$
$1,2 \quad 2,2 \quad \quad 6,2$
$\vdots \quad \vdots \quad \quad \vdots$
$1,6 \quad 2,6 \quad \quad 6,6$

b) $2,6$
$3,5$
$4,4$
$5,3$
$6,2$

c)
$P(\text{sum is } 8) = \dfrac{5}{36}$

**2.** (3pts)  Suppose the house odds on the horse Seven Legs are 3 to 7. If you think the horse's chances of winning are 70%, is this a fair bet?

odds against $= \dfrac{1 - 0.7}{0.7} = \dfrac{0.3}{0.7} = \dfrac{3}{7}$, equal to house odds of $\dfrac{3}{7}$

It is a fair bet.

**3.** (5pts)  In a class of 27 children, 13 play soccer, 8 play basketball and 18 play at least one of the sports. If a child is randomly selected, what is the probability that
a) it plays soccer and basketball?
b) it plays neither sport?

a) $P(\text{soccer or basket}) = P(\text{soccer}) + P(\text{basket}) - P(\text{soccer and basket})$

$\dfrac{18}{27} = \dfrac{13}{27} + \dfrac{8}{27} - P(\text{soccer and basket})$

$P(\text{soccer and basket}) = \dfrac{13}{27} + \dfrac{8}{27} - \dfrac{18}{27} = \dfrac{3}{27} = \dfrac{1}{9}$

b) $P(\text{neither sport}) = 1 - P(\text{soccer or basket})$

$= 1 - \dfrac{18}{27} = \dfrac{27 - 18}{27} = \dfrac{9}{27} = \dfrac{1}{3}$

**4.** (2pts) If a random card is drawn from a deck, the odds against this card being a spade are ___3___ to ___1___.

$39$ to $13 \neq 3$ to $1$

**5.** (7pts) A bag has 8 white and 10 black balls. If two balls are pulled from the bag without looking, what is the probability
a) that they are both white?
b) that the second ball is black, given that the first one is white?
c) that the second ball is black?

a) $P(\text{1st white and 2nd white}) = P(\text{1st white}) \cdot P(\text{2nd white} \mid \text{1st white})$

$$= \frac{8}{18} \cdot \frac{7}{17} = \frac{14}{146}$$

b) $P(\text{2nd black} \mid \text{1st is white}) = \frac{10}{17}$

c) $P(\text{2nd black}) = \frac{10}{18} = \frac{5}{9}$  (same as pulling just one ball out)

**6.** (7pts) Suppose a multiple-choice exam has four possible answers for each question, only one of them correct. You get 5 points for each correct answer, lose 2 points for each incorrect answer and get nothing if you leave the question unanswered.
a) What is the expected point value of a random guess on this exam?
b) What is the expected point value if you can eliminate one of the answers as incorrect and choose a random answer from the remaining three?
c) Assuming you can always eliminate one answer and choose a random answer from the remaining ones, how many points would you expect to get on a test with 50 questions?

a)

outcomes ⟨ get 5 pts / lose 2 pts

expected value =

$= 5 \cdot P(\text{correct}) + (-2) P(\text{incorrect})$

$= 5 \cdot \frac{1}{4} + (-2) \cdot \frac{3}{4} = \frac{5}{4} - \frac{6}{4} = -\frac{1}{4}$

expect to lose 1/4 pt on every question

b) expected value $= 5 \cdot \frac{1}{3} + (-2) \frac{2}{3}$

$= \frac{5}{3} - \frac{4}{3} = \frac{1}{3}$

c) expect to get $50 \cdot \frac{1}{3}$ pts

$= 16 \frac{2}{3}$ points