

Spring '07/MAT 117/Exam 1 Name:

Show all your work.

$$F = P(1+rt) \quad F = P \left(1 + \frac{r}{n}\right)^{nt} \quad F = D \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = R \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad APY = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (4pts) Lou deposits a certain amount of money in an account bearing 4.23% simple interest. After 8 months he withdraws \$462.69. How much did he deposit?

2. (5pts) True story: a short-term loan company advertises on its website that one can get a \$400 loan from them that is repaid after 14 days with \$470. What simple annual interest rate are they charging?

3. (6pts) What is a better deal on a certificate of deposit:

- a) an account earning 3.17%, compounded weekly, or
- b) an account earning 3.15%, compounded daily?

4. (6pts) On February 5th, 1997 the stock of PepsiCo, Inc. closed at \$25.26 per share. On February 5th, 2007 it closed at \$64.83 per share. Find the annual compound interest rate that this growth corresponds to.

5. (6pts) Barack would like to use some of his own money to finance a political campaign. How much should he deposit weekly into an account bearing 5%, compounded weekly, if he would like to have \$1,000,000 in a year-and-a-half?

6. (15pts) PC and Mac have spent a lot of time together lately, so they decided to jointly buy a plasma TV. The biggest they could find was a 103-inch retailing for \$70,000 (I kid you not!), for which they have secured a 5-year loan at 8.49%, compounded monthly.

- a) What is their monthly payment on the loan?
- b) How much do they owe after 4 years?
- c) What are their total payments over the course of the loan?
- d) Which portion of their 1st payment goes toward interest, and which towards the principal?

7. (8pts) If you deposit \$400 every quarter in an account bearing 7.26%, compounded quarterly, how long will it take until you have \$10,000 in the account?

Bonus. (5pts) A couple of newlyweds took out a 15-year, \$234,000 loan to finance their new home. The interest rate on this loan is 5.73% compounded monthly, making their monthly payment \$1940.65. How long will it be until they owe half the amount on the loan? *Hint: only one formula is needed.*

$$\frac{a}{b} = \frac{1-P(E)}{P(E)} \quad P(E) = \frac{b}{a+b} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

1. (2pts) If 87% of households in a town earn more than \$20,000 in a year, what is the probability that a random selected household earns less than \$20,000 in a year?

2. (2pts) During the first two years of college, you took 80 exams, and on 11 of them there was a question that completely surprised you. What is the empirical probability of receiving a question on an exam that completely surprises you?

3. (7pts) A coin is tossed three times.

- List all the equally likely outcomes of this experiment. How many are there?
- List the outcomes on which exactly one head was tossed.
- What is the probability of the experiment resulting in exactly one head tossed?

4. (3pts) If a die is rolled, the odds against getting a 3 or a 5 on the roll are _____ to _____.

5. (3pts) If the probability of finding a quarter on the sidewalk during a daily walk is 10%, what are the odds against finding a quarter on the sidewalk during a daily walk?

6. (8pts) You play the following game, for which there is a \$1 charge: if you get a 3 on a roll of a die, you win \$4.50 (and your \$1 is returned), otherwise you win nothing.

a) What is the expected value for this game of chance?

b) If you play the game 25 times, how much do you expect to win (or lose)?

c) Use expected value to determine whether this game represents a fair bet.

d) Use odds to determine whether this game represents a fair bet.

7. (6pts) In a dealer's lot with 35 cars, 27 have power windows, 13 have power seats and 10 have both one of those features. If a car is randomly selected, what is the probability that

- it has at least one of the features?
- it has neither of the features?

8. (10pts) On any day at a convenience store, there is a 15% chance that it has run out of "Gong" brand of potato chips, and a 65% chance that at least one of the lights has burned out. Assume these two events are independent.

- What is the probability that the store has run out of "Gong" potato chips and has at least one light burnt out?
- What is the probability that the store has all lights working, but has run out of "Gong" potato chips?
- What is the probability that the store has run out of "Gong" potato chips, given that at least one of the lights has burned out?

9. (9pts) Two children are chosen at random from a group of 15 boys and 17 girls.

a) What is the probability that both children are boys?

b) What is the probability that the second child is a girl?

c) What is the probability that the second child is a girl, given that the first child was a girl?

Bonus. (5pts) Jorge and Manuela are separately asked to choose between strawberries and kiwifruit. They will get a snack only if they name different choices. If Jorge names strawberries 70% of the time and kiwifruit 30% of the time, and Manuela names strawberries 65% of the time and kiwifruit 35% of the time, what is the probability that they get a snack?

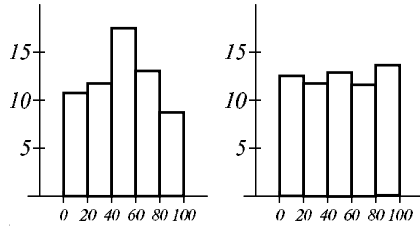
$$\text{angle} = (\text{relative frequency}) \cdot 360^\circ \quad Z = \frac{X - \mu}{\sigma}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \quad \sigma = \sqrt{\frac{f_1(x_1 - \mu)^2 + f_2(x_2 - \mu)^2 + \cdots + f_n(x_n - \mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

1. (9pts) According to the U.S. Bureau of Census, from 1986 to 1995, the percentages of students in grades 10 through 12 who dropped out in a single year were 4.3, 4.1, 4.8, 4.5, 4.0, 4.0, 4.3, 4.2, 5.0, 5.4, given in order of years.
- Find the median dropout rate.
 - Find the mean dropout rate.
 - Find the standard deviation.

2. (3pts) Histograms for two data sets, which have the same mean $\mu = 59$, are shown. Which of the data sets will have a greater standard deviation and why?



3. (14pts) The frequency distribution of the minimum age to receive an unrestricted driver's license in each of the 50 states and the District of Columbia is shown in the table.

- Find the relative frequencies for each class.
- Find the appropriate angles and draw a pie chart for the data.
- Find the median of the data.
- Find the mean of the data.
- Find the standard deviation of the data.

Minimum Age (yrs)	Number of states	Relative frequency	Angle
15	3		
16	28		
17	12		
18	8		

4. (7pts) This semester on exam 2, my Calculus 1 class achieved scores summarized in the table below. Do the following:

- Draw a bar graph for the data.
- Enter a representative value for each interval.
- Estimate the mean of data.

Range	Frequency	Rep. value
90–100	2	
80–89	8	
70–79	5	
60–69	2	
0–60	8	

5. (10pts) Compute the following probabilities for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

a) $P(-0.3 \leq Z < 0.15)$

b) $P(Z \leq 1.3)$

6. (7pts) Based on the U.S. Bureau of the Census statistics, the ages of women who bore a child in 1992 were roughly normally distributed with mean 27.5 years old and a standard deviation of 6 years. Of the women who bore a child in 1992, what is the percentage that were between the ages of 18 and 22?

Bonus. (5pts) Referring to the above problem, what is the age that falls at the 30th percentile of the ages of the women who bore children in 1992.

1. (5pts) The members of a marching band are voting to choose what time they want to hold their evening rehearsals. The results of an approval election are in the table. Which option wins using the approval method?

Number of votes:	28	33	40	19	9	24
6PM	X			X	X	
7PM		X		X		X
8PM			X		X	X

2. (6pts) Suppose there are 75 votes cast in an election between three candidates, decided by plurality. After the first 50 votes are counted, the tally is Xie 14, Vasilev 16, Bergthold 20.
- a) What is the minimal number of remaining votes Bergthold needs to be assured of a win?
- b) What is the minimal number of remaining votes Xie needs to be assured of a win?
- Justify your answers.

3. (14pts) Fans of a circus clown act are voting for their favorite clown. The preference rankings are as follows:

Number of votes:	11	12	7	20	14
Clippy	1	1	2	3	4
Snippy	2	3	1	4	2
Trippy	4	2	4	1	3
Flippy	3	4	3	2	1

- Which clown wins using the plurality method?
- Which clown wins using the plurality method, followed by a runoff of the two top finishers?
- In the plurality with runoff election, can the seven fans who ranked Snippy first obtain a preferable outcome if they voted strategically, assuming all the other fans voted as shown in the table?

4. (5pts) If 124 votes are cast, what is the smallest number of votes a winning candidate can have in a five-candidate race that is decided by plurality? Justify your answer.

5. (15pts) A group of students are devising a prank to play on a professor. The options are: steal couch from professor's office, then send photos of couch in various places around town¹; attach unflattering photos of professor to their car in a place where the professor is unlikely to see it, but others will; and break into professor's office and leave a dozen day-old chicks on their desk. The preference rankings for the prank are below:

Number of votes:	2	3	3	2	5
Couch kidnap	1	1	2	3	2
Photos on car	2	3	1	1	3
Day-old chicks	3	2	3	2	1

- a) Which prank is the Condorcet winner, if any?
- b) Which prank wins using the Borda method?
- c) Perform the check on the sum of Borda points.
- d) Can the two students that voted "couch kidnap" the last obtain a preferable outcome if they voted strategically using the Borda method?

¹actually performed on a math professor

6. (5pts) Suppose three candidates are running in an election decided by plurality with a runoff between the two top finishers. If the results of the first ballot are Lewis 135, Junghenn 115, Albert 143, what percentage of Junghenn supporters need to vote for Lewis in order for Lewis to win the election?

Bonus. (5pts) Devise a scenario with three candidates that shows that plurality with runoff does not satisfy the property of independence from irrelevant alternatives. (Recall that the property says: if A wins over B in a two-candidate race, then in a race with any additional candidates, B cannot win.) Your answer should be a table with preference rankings for the three candidates, along with a tally of votes showing the property above is violated.

$$\text{angle} = (\text{relative frequency}) \cdot 360^\circ \quad Z = \frac{X - \mu}{\sigma}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$$

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1. (4pts) If 99 votes are cast, what is the smallest number of votes a winning candidate can have in a three-candidate race that is decided by plurality? Justify your answer.

2. (5pts) Compute the following probability for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

$$P(.34 \leq Z \leq 1.19) =$$

3. (13pts) A group of opera critics are choosing their favorite present-day soprano. The preference rankings for three sopranos are below:

Number of votes:	3	7	4	3	5	4
Anna Netrebko	1	1	2	3	2	3
Angela Gheorghiu	2	3	1	1	3	2
Renée Fleming	3	2	3	2	1	1

- a) Which soprano wins using the plurality method?
- b) Which soprano wins using the Plurality with runoff method?
- c) Which soprano wins using the Borda method?
- d) Perform the check on the sum of Borda points.
- e) In the Borda method, can the three critics who voted Gheorghiu first and Fleming second obtain a preferable outcome if they voted strategically?

4. (12pts) There will be 67 total solar eclipses on Earth from 2001 to 2100. The duration of solar eclipses varies with the frequency distribution of the durations shown below.

- a) Find the median duration.
- b) Find the mean duration.
- c) Find the standard deviation.

Duration (min)	Frequency
2	19
3	19
4	14
5	8
6	7

5. (7pts)

- a) If one card is drawn from a deck of cards, what is the probability that it is an ace?
- b) If two cards are drawn from a deck of cards, what is the probability that the second card is a spade, given that the first card was a diamond?
- c) If two cards are drawn from a deck of cards, what is the probability that both are diamonds?

6. (8pts) A spinner can stop in one of four equal-sized fields labeled A, B, C, D. Using the spinner, you play the following game, for which there is a \$1 charge. If you get a B on a spin, you win \$3.00 (and your \$1 is returned), otherwise, you win nothing.

a) What is the expected value for this game of chance?

b) If you play the game 10 times, how much do you expect to win (or lose)?

c) What are the house odds on this bet? What are the true odds against the spinner stopping on B?

d) Use odds to determine whether this game represents a fair bet.

7. (4pts) Sam will take two final exams on the same day, one in psychology, one in biology. The probability there is a surprise question on the psychology exam is 15%, and the probability there is a surprise question on the biology exam is 20%. Assuming the writers of the two exams work independently, what is the probability that Sam gets a surprise question on both exams that day?

8. (4pts) If \$2,000 is deposited into an account bearing 4.53%, compounded monthly, how much is in the account after two-and-a-half years?

9. (5pts) Melissa would like to save money to buy a car for \$17,000. How much should she deposit every week into an account bearing 5%, compounded weekly, in order to save up for the car in three years?

10. (8pts) Angelina Jolie is building an orphanage in Vietnam for which she needs to borrow \$1,500,000. Suppose she can get a 15-year loan with interest rate 5.58%, compounded monthly.

- a) What is her monthly payment?
- b) What is the balance on the loan after 5 years?

Bonus. (7pts) A 1991 survey done by the U.S. Bureau of Justice shows that the age of inmates in state prisons was approximately normally distributed with mean 32.4 years and standard deviation 9.9 years. What is the percentage of inmates between the ages of 30 and 35? (Draw a picture of the normal distribution.)