Spring '07/MAT 117/Final Exam **Name:** Solution *Show all your work.*

angle = (relative frequency) $\cdot$ 360° $\quad Z = \dfrac{X-\mu}{\sigma}$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad \sigma = \sqrt{\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$$

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} \qquad \sigma = \sqrt{\frac{f_1(x_1-\mu)^2 + f_2(x_2-\mu)^2 + \cdots + f_n(x_n-\mu)^2}{f_1 + f_2 + \cdots + f_n}}$$

$\frac{a}{b} = \frac{1-P(E)}{P(E)} \quad P(E) = \frac{b}{a+b} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$F = P(1+rt) \quad F = P\left(1+\frac{r}{n}\right)^{nt} \quad F = D\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}} \quad P = R\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad APY = \left(1+\frac{r}{n}\right)^{n}-1$

1. (4pts) If 99 votes are cast, what is the smallest number of votes a winning candidate can have in a three-candidate race that is decided by plurality? Justify your answer.

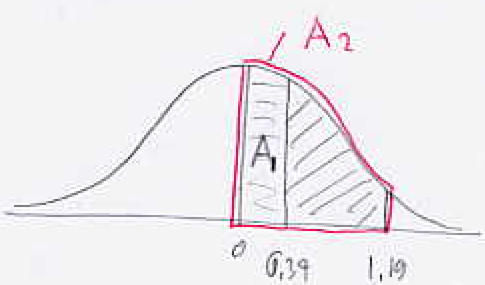
$99 \div 3 = 33$ If votes are evenly distributed among the candidates, each one gets 33.

To win, one candidate needs one more vote, so 34.

2. (5pts) Compute the following probability for a standard normal distribution. Draw a picture showing which area you are computing — shading is a good thing!

$P(.34 \leq Z \leq 1.19) = A_2 - A_1 = 0.3830 - 0.1331$

$= 0.2499$



3. (13pts) A group of opera critics are choosing their favorite present-day soprano. The preference rankings for three sopranos are below:

Number of votes:	3	7	4	3	5	4
Anna Netrebko	1	1	2	3	2	3
Angela Gheorghiu	2	3	1	1	3	2
Renée Fleming	3	2	3	2	1	1

= 26 votes

a) Which soprano wins using the plurality method?
b) Which soprano wins using the Plurality with runoff method?
c) Which soprano wins using the Borda method?
d) Perform the check on the sum of Borda points.
e) In the Borda method, can the three critics who voted Gheorghiu first and Fleming second obtain a preferable outcome if they voted strategically?

a) AN 3+7 = 10 wins
AG 4+3 = 7
RF 5+4 = 9

b) Runoff btw AN, RF
AN 10 + 4 = 14
RF 9 + 3 = 12

c) AN $10 \cdot 3 + 9 \cdot 2 + 7 \cdot 1 = 55$ wins
AG $7 \cdot 3 + 7 \cdot 2 + 12 \cdot 1 = 47$
RF $9 \cdot 3 + 10 \cdot 2 + 7 \cdot 1 = 54$
156

d) 26 voters × 6 = 156

e)

		– contribution of 3	total w/o contribution	new contribution	
AN	55	– 3	52	+3	55
AG	47	– 9	38	+6	44
RF	54	– 6	48	+5	57 wins

if the three vote AN 3
AG 2
RF 1

38th

4. (12pts) There will be 67 total solar eclipses on Earth from 2001 to 2100. The duration of solar eclipses varies with the frequency distribution of the durations shown below.

a) Find the median duration.
b) Find the mean duration.
c) Find the standard deviation.

Duration (min)	Frequency
2	19
3	19
4	14
5	8
6	7
	67

a) $\underbrace{2,\dots,2}_{19}, \underbrace{3,\dots,3}_{19}, \underbrace{4,\dots,4}_{14}, \underbrace{5,\dots,5}_{8}, \underbrace{6,\dots,6}_{7}$

Need middle number, $67 \div 2 = 33.5$ so 34th

34th is among the 3's so median = 3

b) $$\mu = \frac{19\cdot 2 + 19\cdot 3 + 14\cdot 4 + 8\cdot 5 + 7\cdot 6}{67} = \frac{233}{67} \approx 3.48$$

c) $$\sigma^2 = \frac{19(2-3.48)^2 + 19(3-3.48)^2 + 14(4-3.48)^2 + 8(5-3.48)^2 + 7(6-3.48)^2}{67}$$

$$= \frac{112.71}{67} = 1.68\ldots$$

$$\sigma = \sqrt{1.68} \approx 1.297$$

5. (7pts)
a) If one card is drawn from a deck of cards, what is the probability that it is an ace?
b) If two cards are drawn from a deck of cards, what is the probability that the second card is a spade, given that the first card was a diamond?
c) If two cards are drawn from a deck of cards, what is the probability that both are diamonds?

a) $P(\text{ace}) = \frac{4}{52} = \frac{1}{13} = 0.0769$

b) $P(\text{2nd is spade} \mid \text{1st is diamond}) = \frac{13}{51} = 0.2549$

c) $P(\text{1st diamond and 2nd diamond}) = P(\text{1st diamond}) \cdot P(\text{2nd diamond} \mid \text{1st diamond})$

$$= \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{4} \cdot \frac{12}{51} = \frac{3}{51} = 0.0588$$

6. (8pts) A spinner can stop in one of four equal-sized fields labeled A, B, C, D. Using the spinner, you play the following game, for which there is a \$1 charge. If you get a B on a spin, you win \$3.00 (and your \$1 is returned), otherwise, you win nothing.
a) What is the expected value for this game of chance?
b) If you play the game 10 times, how much do you expect to win (or lose)?
c) What are the house odds on this bet? What are the true odds against the spinner stopping on B?
d) Use odds to determine whether this game represents a fair bet.

a) expected value (net win) $= 3 \cdot P(\text{win}) + (-1) P(\text{lose}) = 3 \cdot \frac{1}{4} + (-1)\frac{3}{4} = 0$

b) You would expect to win $10 \cdot 0 = 0$

c) House odds are 3 to 1

True odds are 3 to 1

d) This is a fair bet because house odds = true odds against

(or: it is a fair bet since expected value = 0)

7. (4pts) Sam will take two final exams on the same day, one in psychology, one in biology. The probability there is a surprise question on the psychology exam is 15%, and the probability there is a surprise question on the biology exam is 20%. Assuming the writers of the two exams work independently, what is the probability that Sam gets a surprise question on both exams that day?

P(suprise on exam 1 and surprise on exam 2) = [events are independent]

= P(surprise on exam 1) · P(surprise on exam 2)

= $0.15 \cdot 0.20 = 0.03$

8. (4pts) If \$2,000 is deposited into an account bearing 4.53%, compounded monthly, how much is in the account after two-and-a-half years?

$$F = 2000\left(1+\frac{0.0453}{12}\right)^{12\cdot 2.5} = 2000\cdot(1.003775)^{30} =$$

$$= 2000 \cdot 1.119673\ldots$$

$$= 2239.35$$

9. (5pts) Melissa would like to save money to buy a car for \$17,000. How much should she deposit every week into an account bearing 5%, compounded weekly, in order to save up for the car in three years?

$$17000 = D\,\frac{\left(1+\frac{0.05}{52}\right)^{52\cdot 3}-1}{\frac{0.05}{52}}$$

$$17000 = D\,\frac{(1.000961\ldots)^{156}-1}{0.000961\ldots}$$

$$17000 = D\cdot 168.22\ldots$$

$$D = \frac{17000}{168.22\ldots} \approx 101.06$$ weekly deposit

10. (8pts) Angelina Jolie is building an orphanage in Vietnam for which she needs to borrow \$1,500,000. Suppose she can get a 15-year loan with interest rate 5.58%, compounded monthly.
a) What is her monthly payment?
b) What is the balance on the loan after 5 years?

a) $$1{,}500{,}000 = R\,\frac{1-\left(1+\frac{0.0558}{12}\right)^{-12\cdot 15}}{\frac{0.0558}{12}}$$

$$1{,}500{,}000 = R\,\frac{1-(1.00465)^{-180}}{0.00465}$$

$$1{,}500{,}000 = R\cdot 121.75$$

$$R = \frac{1{,}500{,}000}{121.75\ldots} = 12{,}320.02$$

b) Balance = present value of remaining 10 years of payments

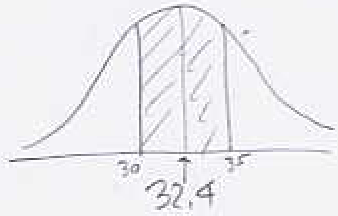
$$P = 12{,}320.02\cdot\frac{1-\left(1+\frac{0.0558}{12}\right)^{-12\cdot 10}}{\frac{0.0558}{12}}$$

$$= 12{,}320.02\cdot\frac{1-(1.00465)^{-120}}{0.00465}$$

$$= 12{,}320.02\cdot 91.81\ldots$$

$$= 1{,}131{,}075.20$$

Bonus. (7pts) A 1991 survey done by the U.S. Bureay of Justice shows that the age of inmates in state prisons was approximately normally distributed with mean 32.4 years and standard deviation 9.9 years. What is the percentage of inmates betwen the ages of 30 and 35? (Draw a picture of the normal distribution.)



$$P(30 \le X \le 35)$$

$$= P\left(\frac{30-32.4}{9.9} \le Z \le \frac{35-32.4}{9.9}\right)$$

$$= P(-0.24 \le Z \le 0.26)$$

$$= A_1 + A_2$$

$$= 0.0948 + 0.1026$$

$$= 0.1974$$