

1. (5pts) The members of a marching band are voting to choose what time they want to hold their evening rehearsals. The results of an approval election are in the table. Which option wins using the approval method?

Number of votes:	28	33	40	19	9	24
6PM	X			X	X	
7PM		X		X		X
8PM			X		X	X

Approval votes: 6PM: $28 + 19 + 9 = 56$

7PM: $33 + 19 + 24 = 76$ wins

8PM: $40 + 9 + 24 = 73$

2. (6pts) Suppose there are 75 votes cast in an election between three candidates, decided by plurality. After the first 50 votes are counted, the tally is Xie 14, Vasilev 16, Bergthold 20.

- a) What is the minimal number of remaining votes Bergthold needs to be assured of a win?
b) What is the minimal number of remaining votes Xie needs to be assured of a win?

Justify your answers.

a) X 14
V 16
B 20

In a worst-case scenario for B,

V gets 4 votes to catch up,

$$25 - 4 = 21$$

B needs 11 of remaining votes to be sure of a win,

b) X needs 6 votes to catch up.

Of the remaining $25 - 6 = 19$,

he needs 10 to be assured of a win.

$$10 + 6 = 16 \text{ votes}$$

3. (14pts) Fans of a circus clown act are voting for their favorite clown. The preference rankings are as follows:

Number of votes:	11	12	7	20	14
Clippy	1	1	2	3	4
Snippy	2	3	1	4	2
Trippy	4	2	4	1	3
Flippy	3	4	3	2	1

- a) Which clown wins using the plurality method?
 b) Which clown wins using the plurality method, followed by a runoff of the two top finishers?
 c) In the plurality with runoff election, can the ~~five~~^{seven} fans who ranked Snippy first obtain a preferable outcome if they voted strategically, assuming all the other fans voted as shown in the table?

a) C $11+12=23$ wins
 S 7
 T 20
 F 14

c) Assigning first ranking to Clippy will not help, since he already makes it into runoff and loses runoff.

If the 7 fans rank

C	2
S	3
T	4
F	1

b) C $23+7=30$
 T $20+14=34$ wins

results of plurality election are:

C	23
S	0
T	20
F	21

so Clippy and Flippy are in runoff.

Runoff scores are

C 23

better outcome

F $21+20=41$

for the seven fans,

4. (5pts) If 124 votes are cast, what is the smallest number of votes a winning candidate can have in a five-candidate race that is decided by plurality? Justify your answer.

$124 \div 5 = 24$, remainder 4

If votes are evenly distributed (24 each, 4 remain)
 a candidate needs $24+2$ to have more than another candidate, so 26.

5. (15pts) A group of students are devising a prank to play on a professor. The options are: steal couch from professor's office, then send photos of couch in various places around town¹; attach unflattering photos of professor to their car in a place where the professor is unlikely to see it, but others will; and break into professor's office and leave a dozen day-old chicks on their desk. The preference rankings for the prank are below:

	Number					
Percentage of votes:	2	3	3	2	5	= 15 voters
Couch kidnap	1	1	2	3	2	
Photos on car	2	3	1	1	3	
Day-old chicks	3	2	3	2	1	

- Which prank is the Condorcet winner, if any?
- Which prank wins using the Borda method?
- Perform the check on the sum of Borda points.
- Can the two students that voted "couch kidnap" the last obtain a preferable outcome if they voted strategically?

a) C $2+3+5 = 10$ wins

P $3+2 = 5$

C $2+3+3 = 8$ wins

D $5+2 = 7$

P $3+2+2 = 7$

D $5+3 = 8$ wins

Couch kidnap is the Condorcet winner

b) C $5 \cdot 3 + 8 \cdot 2 + 2 \cdot 1 = 33$ wins

P $5 \cdot 3 + 2 \cdot 2 + 8 \cdot 1 = 27$

D $5 \cdot 3 + 5 \cdot 2 + 5 \cdot 1 = 30$
 $\underline{\hspace{1cm}}$
 90

c) $90 = 15 \cdot 6$

d) Suppose the two students ranked

as
 C: 3
 P: 2
 D: 1
 instead

score		score w/o their contribution	altered contribution	new score
33	-2	31	+2	33
27	-6	21	+4	25
30	-4	26	+6	32

↑
 contribution of the 2

Even with strategic voting, cannot achieve a win by another option

¹actually performed on a math professor

6. (5pts) Suppose three candidates are running in an election decided by plurality with a runoff between the two top finishers. If the results of the first ballot are Lewis 135, Junghenn 115, Albert 143, what percentage of Junghenn supporters need to vote for Lewis in order for Lewis to win the election?

L 135 L needs 8 votes to catch up with A.
 J 115 Of the remaining $115 - 8 = 107$ votes,
 A 143 L needs to get 54 of the 107 votes to win.
 Thus, L needs 62 votes, or $\frac{62}{115} = 53.9\%$ of J's votes.

Bonus. (5pts) Devise a scenario with three candidates that shows that plurality with runoff does not satisfy the property of independence from irrelevant alternatives. (Recall that the property says: if A wins over B in a two-candidate race, then in a race with any additional candidates, B cannot win.) Your answer should be a table with preference rankings for the three candidates, along with a tally of votes showing the property above is violated.

	2	3	4
A	1	2	3
B	2	3	1
C	3	1	2

plurality: A 2
 w/3 B 4
 candidate C 3

} causing runoff: B 4+2 B wins
 C 3 runoff.

plurality with just A, B

A 5
 B 4